30

INDUCTANCE

30.1. **IDENTIFY** and **SET UP:** Apply Eq. (30.4).

EXECUTE: **(a)** $|\mathcal{E}_2| = M\left|\dfrac{di_1}{dt}\right| = (3.25\times10^{-4}\ \text{H})(830\ \text{A/s}) = 0.270\ \text{V};$ yes, it is constant.

(b) $|\mathcal{E}_1| = M\left|\dfrac{di_2}{dt}\right|;$ M is a property of the pair of coils so is the same as in part (a). Thus $|\mathcal{E}_1| = 0.270\ \text{V}$.

EVALUATE: The induced emf is the same in either case. A constant di/dt produces a constant emf.

30.2. **IDENTIFY:** $\mathcal{E}_1 = M\left|\dfrac{\Delta i_2}{\Delta t}\right|$ and $\mathcal{E}_2 = M\left|\dfrac{\Delta i_1}{\Delta t}\right|$. $M = \left|\dfrac{N_2\Phi_{B2}}{i_1}\right|$, where Φ_{B2} is the flux through one turn of the second coil.

SET UP: M is the same whether we consider an emf induced in coil 1 or in coil 2.

EXECUTE: **(a)** $M = \dfrac{\mathcal{E}_2}{|\Delta i_1/\Delta t|} = \dfrac{1.65\times10^{-3}\ \text{V}}{0.242\ \text{A/s}} = 6.82\times10^{-3}\ \text{H} = 6.82\ \text{mH}$

(b) $\Phi_{B2} = \dfrac{Mi_1}{N_2} = \dfrac{(6.82\times10^{-3}\ \text{H})(1.20\ \text{A})}{25} = 3.27\times10^{-4}\ \text{Wb}$

(c) $\mathcal{E}_1 = M\left|\dfrac{\Delta i_2}{\Delta t}\right| = (6.82\times10^{-3}\ \text{H})(0.360\ \text{A/s}) = 2.46\times10^{-3}\ \text{V} = 2.46\ \text{mV}$

EVALUATE: We can express M either in terms of the total flux through one coil produced by a current in the other coil, or in terms of the emf induced in one coil by a changing current in the other coil.

30.3. **IDENTIFY:** A coil is wound around a solenoid, so magnetic flux from the solenoid passes through the coil.
SET UP: Example 30.1 shows that the mutual inductance for this configuration of coils is
$M = \dfrac{\mu_0 N_1 N_2 A}{l}$, where l is the length of coil 1.

EXECUTE: Using the formula for M gives

$$M = \frac{(4\pi\times10^{-7}\ \text{Wb/m}\cdot\text{A})(800)(50)\pi(0.200\times10^{-2}\ \text{m})^2}{0.100\ \text{m}} = 6.32\times10^{-6}\ \text{H} = 6.32\ \mu\text{H}.$$

EVALUATE: This result is a physically reasonable mutual inductance.

30.4. **IDENTIFY:** Changing flux from one object induces an emf in another object.
(a) SET UP: The magnetic field due to a solenoid is $B = \mu_0 nI$.

EXECUTE: The above formula gives

$$B_1 = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(300)(0.120\ \text{A})}{0.250\ \text{m}} = 1.81\times10^{-4}\ \text{T}$$

The average flux through each turn of the inner solenoid is therefore

$$\Phi_B = B_1 A = (1.81\times10^{-4}\ \text{T})\pi(0.0100\ \text{m})^2 = 5.68\times10^{-8}\ \text{Wb}$$

(b) SET UP: The flux is the same through each turn of both solenoids due to the geometry, so

$$M=\frac{N_2\Phi_{B,2}}{i_1}=\frac{N_2\Phi_{B,1}}{i_1}$$

EXECUTE: $M=\dfrac{(25)(5.68\times10^{-8}\text{ Wb})}{0.120\text{ A}}=1.18\times10^{-5}\text{ H}$

(c) SET UP: The induced emf is $\mathcal{E}_2=-M\dfrac{di_1}{dt}$.

EXECUTE: $\mathcal{E}_2=-(1.18\times10^{-5}\text{ H})(1750\text{ A/s})=-0.0207\text{ V}$

EVALUATE: A mutual inductance around 10^{-5} H is not unreasonable.

30.5. **IDENTIFY** and **SET UP:** Apply Eq. (30.5).

EXECUTE: **(a)** $M=\dfrac{N_2\Phi_{B2}}{i_1}=\dfrac{400(0.0320\text{ Wb})}{6.52\text{ A}}=1.96\text{ H}$

(b) $M=\dfrac{N_1\Phi_{B1}}{i_2}$ so $\Phi_{B1}=\dfrac{Mi_2}{N_1}=\dfrac{(1.96\text{ H})(2.54\text{ A})}{700}=7.11\times10^{-3}\text{ Wb}$

EVALUATE: M relates the current in one coil to the flux through the other coil. Eq. (30.5) shows that M is the same for a pair of coils, no matter which one has the current and which one has the flux.

30.6. **IDENTIFY:** One toroidal solenoid is wound around another, so the flux of one of them passes through the other.

SET UP: $B_1=\dfrac{\mu_0N_1i_1}{2\pi r}$ for a toroidal solenoid, $M=\dfrac{N_2|\Phi_{B2}|}{i_1}$.

EXECUTE: **(a)** $B_1=\dfrac{\mu_0N_1i_1}{2\pi r}$. For each turn in the second solenoid the flux is $|\Phi_{B2}|=B_1A=\dfrac{\mu_0N_1i_1A}{2\pi r}$.

Therefore $M=\dfrac{N_2|\Phi_{B2}|}{i_1}=\dfrac{\mu_0N_1N_2A}{2\pi r}$.

(b) $M=\dfrac{\mu_0N_1N_2A}{2\pi r}=(2\times10^{-7}\text{ T}\cdot\text{m/A})\dfrac{(500)(300)(0.800\times10^{-4}\text{ m}^2)}{0.100\text{ m}}=2.40\times10^{-5}\text{ H}=24.0\ \mu\text{H}$.

EVALUATE: This result is a physically reasonable mutual inductance.

30.7. **IDENTIFY:** We can relate the known self-inductance of the toroidal solenoid to its geometry to calculate the number of coils it has. Knowing the induced emf, we can find the rate of change of the current.

SET UP: Example 30.3 shows that the self-inductance of a toroidal solenoid is $L=\dfrac{\mu_0N^2A}{2\pi r}$. The voltage across the coil is related to the rate at which the current in it is changing by $\mathcal{E}=L\left|\dfrac{di}{dt}\right|$.

EXECUTE: **(a)** Solving $L=\dfrac{\mu_0N^2A}{2\pi r}$ for N gives

$$N=\sqrt{\frac{2\pi rL}{\mu_0A}}=\sqrt{\frac{2\pi(0.0600\text{ m})(2.50\times10^{-3}\text{ H})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.00\times10^{-4}\text{ m}^2)}}=1940\text{ turns.}$$

(b) $\left|\dfrac{di}{dt}\right|=\dfrac{\mathcal{E}}{L}=\dfrac{2.00\text{ V}}{2.50\times10^{-3}\text{ H}}=800\text{ A/s}$.

EVALUATE: The inductance is determined solely by how the coil is constructed. The induced emf depends on the rate at which the current through the coil is changing.

30.8. **IDENTIFY:** A changing current in an inductor induces an emf in it.

(a) SET UP: The self-inductance of a toroidal solenoid is $L=\dfrac{\mu_0N^2A}{2\pi r}$.

EXECUTE: $L=\dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(500)^2(6.25\times10^{-4}\text{ m}^2)}{2\pi(0.0400\text{ m})}=7.81\times10^{-4}\text{ H}$

(b) SET UP: The magnitude of the induced emf is $\mathcal{E} = L\dfrac{di}{dt}$.

EXECUTE: $\mathcal{E} = (7.81\times10^{-4}\ \text{H})\left(\dfrac{5.00\ \text{A} - 2.00\ \text{A}}{3.00\times10^{-3}\ \text{s}}\right) = 0.781\ \text{V}$

(c) The current is decreasing, so the induced emf will be in the same direction as the current, which is from a to b, making b at a higher potential than a.

EVALUATE: This is a reasonable value for self-inductance, in the range of a mH.

30.9. **IDENTIFY:** $\mathcal{E} = L\left|\dfrac{\Delta i}{\Delta t}\right|$ and $L = \dfrac{N\Phi_B}{i}$.

SET UP: $\dfrac{\Delta i}{\Delta t} = 0.0640\ \text{A/s}$

EXECUTE: **(a)** $L = \dfrac{\mathcal{E}}{|\Delta i/\Delta t|} = \dfrac{0.0160\ \text{V}}{0.0640\ \text{A/s}} = 0.250\ \text{H}$

(b) The average flux through each turn is $\Phi_B = \dfrac{Li}{N} = \dfrac{(0.250\ \text{H})(0.720\ \text{A})}{400} = 4.50\times10^{-4}\ \text{Wb}$.

EVALUATE: The self-induced emf depends on the rate of change of flux and therefore on the rate of change of the current, not on the value of the current.

30.10. **IDENTIFY:** Combine the two expressions for L: $L = N\Phi_B/i$ and $L = \mathcal{E}/|di/dt|$.

SET UP: Φ_B is the average flux through one turn of the solenoid.

EXECUTE: Solving for N we have $N = \mathcal{E}i/\Phi_B|di/dt| = \dfrac{(12.6\times10^{-3}\ \text{V})(1.40\ \text{A})}{(0.00285\ \text{Wb})(0.0260\ \text{A/s})} = 238$ turns.

EVALUATE: The induced emf depends on the time rate of change of the total flux through the solenoid.

30.11. **IDENTIFY** and **SET UP:** Apply $|\mathcal{E}| = L|di/dt|$. Apply Lenz's law to determine the direction of the induced emf in the coil.

EXECUTE: **(a)** $|\mathcal{E}| = L|di/dt| = (0.260\ \text{H})(0.0180\ \text{A/s}) = 4.68\times10^{-3}\ \text{V}$

(b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the + terminal at a.

EVALUATE: The induced emf is directed so as to oppose the decrease in the current.

30.12. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$.

SET UP: The induced emf points from low potential to high potential across the inductor.

EXECUTE: **(a)** The induced emf points from b to a, in the direction of the current. Therefore, the current is decreasing and the induced emf is directed to oppose this decrease.

(b) $|\mathcal{E}| = L|di/dt|$, so $|di/dt| = V_{ab}/L = (1.04\ \text{V})/(0.260\ \text{H}) = 4.00\ \text{A/s}$. In 2.00 s the decrease in i is 8.00 A and the current at 2.00 s is 12.0 A −8.0 A = 4.0 A.

EVALUATE: When the current is decreasing the end of the inductor where the current enters is at the lower potential. This agrees with our result and with Figure 30.6d in the textbook.

30.13. **IDENTIFY:** The inductance depends only on the geometry of the object, and the resistance of the wire depends on its length.

SET UP: $L = \dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: **(a)** $N = \sqrt{\dfrac{2\pi r L}{\mu_0 A}} = \sqrt{\dfrac{(0.120\ \text{m})(0.100\times10^{-3}\ \text{H})}{(2\times10^{-7}\ \text{T}\cdot\text{m/A})(0.600\times10^{-4}\ \text{m}^2)}} = 1.00\times10^3$ turns.

(b) $A = \pi d^2/4$ and $c = \pi d$, so $c = \sqrt{4\pi A} = \sqrt{4\pi(0.600\times10^{-4}\ \text{m}^2)} = 0.02746\ \text{m}$. The total length of the wire is $(1000)(0.02746\ \text{m}) = 27.46\ \text{m}$. Therefore $R = (0.0760\ \Omega/\text{m})(27.46\ \text{m}) = 2.09\ \Omega$.

EVALUATE: A resistance of $2\ \Omega$ is large enough to be significant in a circuit.

30.14. **IDENTIFY:** The changing current induces an emf in the solenoid.

SET UP: By definition of self-inductance, $L = \frac{N\Phi_B}{i}$. The magnitude of the induced emf is $|\mathcal{E}| = L\left|\frac{di}{dt}\right|$.

EXECUTE: $L = \frac{N\Phi_B}{i} = \frac{(800)(3.25\times10^{-3}\text{ Wb})}{2.90\text{ A}} = 0.8966\text{ H}$.

$\left|\frac{di}{dt}\right| = \frac{|\mathcal{E}|}{L} = \frac{7.50\times10^{-3}\text{ V}}{0.8966\text{ H}} = 8.37\times10^{-3}\text{ A/s} = 8.37\text{ mA/s}$.

EVALUATE: An inductance of nearly a henry is rather large. For ordinary laboratory inductors, which are around a few millihenries, the current would have to be changing much faster to induce 7.5 mV.

30.15. **IDENTIFY:** Use the definition of inductance and the geometry of a solenoid to derive its self-inductace.

SET UP: The magnetic field inside a solenoid is $B = \mu_0 \frac{N}{l} i$, and the definition of self-inductance is $L = \frac{N\Phi_B}{i}$.

EXECUTE: **(a)** $B = \mu_0 \frac{N}{l} i$, $L = \frac{N\Phi_B}{i}$, and $\Phi_B = \frac{\mu_0 NAi}{l}$. Combining these expressions gives

$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{l}$.

(b) $L = \frac{\mu_0 N^2 A}{l}$. $A = \pi r^2 = \pi(0.0750\times10^{-2}\text{ m})^2 = 1.767\times10^{-6}\text{ m}^2$.

$L = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(50)^2(1.767\times10^{-6}\text{ m}^2)}{5.00\times10^{-2}\text{ m}} = 1.11\times10^{-7}\text{ H} = 0.111\,\mu\text{H}$.

EVALUATE: This is a physically reasonable value for self-inductance.

30.16. **IDENTIFY** and **SET UP:** The stored energy is $U = \frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P = I^2 R$.

EXECUTE: **(a)** $U = \frac{1}{2}LI^2 = \frac{1}{2}(12.0\text{ H})(0.300\text{ A})^2 = 0.540\text{ J}$

(b) $P = I^2 R = (0.300\text{ A})^2(180\,\Omega) = 16.2\text{ W} = 16.2\text{ J/s}$

EVALUATE: **(c)** No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.17. **IDENTIFY** and **SET UP:** Use Eq. (30.9) to relate the energy stored to the inductance. Example 30.3 gives the inductance of a toroidal solenoid to be $L = \frac{\mu_0 N^2 A}{2\pi r}$, so once we know L we can solve for N.

EXECUTE: $U = \frac{1}{2}LI^2$ so $L = \frac{2U}{I^2} = \frac{2(0.390\text{ J})}{(12.0\text{ A})^2} = 5.417\times10^{-3}\text{ H}$

$N = \sqrt{\frac{2\pi rL}{\mu_0 A}} = \sqrt{\frac{2\pi(0.150\text{ m})(5.417\times10^{-3}\text{ H})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^{-4}\text{ m}^2)}} = 2850.$

EVALUATE: L and hence U increase according to the square of N.

30.18. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a toroidal solenoid is $B = \frac{\mu_0 NI}{2\pi r}$.

EXECUTE: $B = \frac{\mu_0(300)(5.00\text{ A})}{2\pi(0.120\text{ m})} = 2.50\times10^{-3}\text{ T} = 2.50\text{ mT}$

(b) SET UP: The self-inductance of a toroidal solenoid is $L = \frac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)^2(4.00\times10^{-4}\text{ m}^2)}{2\pi(0.120\text{ m})} = 6.00\times10^{-5}\text{ H}$

(c) SET UP: The energy stored in an inductor is $U_L = \frac{1}{2}LI^2$.

EXECUTE: $U_L = \frac{1}{2}(6.00\times10^{-5}\text{ H})(5.00\text{ A})^2 = 7.50\times10^{-4}\text{ J}$

(d) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(2.50\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\text{ J/m}^3$

(e) $u = \dfrac{\text{energy}}{\text{volume}} = \dfrac{\text{energy}}{2\pi rA} = \dfrac{7.50\times10^{-4}\text{ J}}{2\pi(0.120\text{ m})(4.00\times10^{-4}\text{ m}^2)} = 2.49\text{ J/m}^3$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.19. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a solenoid is $B = \mu_0 nI$.

EXECUTE: $B = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(400)(80.0\text{ A})}{0.250\text{ m}} = 0.161\text{ T}$

(b) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(0.161\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 1.03\times10^4\text{ J/m}^3$

(c) SET UP: The total stored energy is $U = uV$.

EXECUTE: $U = uV = u(lA) = (1.03\times10^4\text{ J/m}^3)(0.250\text{ m})(0.500\times10^{-4}\text{ m}^2) = 0.129\text{ J}$

(d) SET UP: The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

EXECUTE: Solving for L and putting in the numbers gives

$$L = \frac{2U}{I^2} = \frac{2(0.129\text{ J})}{(80.0\text{ A})^2} = 4.02\times10^{-5}\text{ H}$$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.20. **IDENTIFY:** Energy $= Pt$. $U = \frac{1}{2}LI^2$.

SET UP: $P = 200\text{ W} = 200\text{ J/s}$

EXECUTE: **(a)** Energy $= (200\text{ W})(24\text{ h})(3600\text{ s/h}) = 1.73\times10^7\text{ J}$

(b) $L = \dfrac{2U}{I^2} = \dfrac{2(1.73\times10^7\text{ J})}{(80.0\text{ A})^2} = 5.41\times10^3\text{ H}$

EVALUATE: A large value of L and a large current would be required, just for one light bulb. Also, the resistance of the inductor would have to be very small, to avoid a large $P = I^2R$ rate of electrical energy loss.

30.21. **IDENTIFY:** The energy density depends on the strength of the magnetic field, and the energy depends on the volume in which the magnetic field exists.

SET UP: The energy density is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: First find the energy density: $u = \dfrac{B^2}{2\mu_0} = \dfrac{(4.80\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 9.167\times10^6\text{ J/m}^3$. The energy U in a volume V is $U = uV = (9.167\times10^6\text{ J/m}^3)(10.0\times10^{-6}\text{ m}^3) = 91.7\text{ J}$.

EVALUATE: A field of 4.8 T is very strong, so this is a high energy density for a magnetic field.

30.22. **IDENTIFY** and **SET UP:** The energy density (energy per unit volume) in a magnetic field (in vacuum) is given by $u = \dfrac{U}{V} = \dfrac{B^2}{2\mu_0}$ (Eq. 30.10).

EXECUTE: **(a)** $V = \dfrac{2\mu_0 U}{B^2} = \dfrac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.60\times10^6\ \text{J})}{(0.600\ \text{T})^2} = 25.1\ \text{m}^3.$

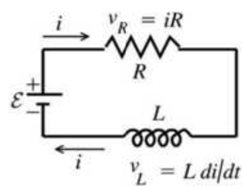
(b) $u = \dfrac{U}{V} = \dfrac{B^2}{2\mu_0}$

$$B = \sqrt{\frac{2\mu_0 U}{V}} = \sqrt{\frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.60\times10^6\ \text{J})}{(0.400\ \text{m})^3}} = 11.9\ \text{T}$$

EVALUATE: Large-scale energy storage in a magnetic field is not practical. The volume in part (a) is quite large and the field in part (b) would be very difficult to achieve.

30.23. **IDENTIFY:** Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by Eq. (30.14).

SET UP: The circuit is sketched in Figure 30.23.



$\dfrac{di}{dt}$ is positive as the current increases from its initial value of zero.

Figure 30.23

EXECUTE: $\mathcal{E} - v_R - v_L = 0$

$\mathcal{E} - iR - L\dfrac{di}{dt} = 0$ so $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$

(a) Initially $(t = 0)$, $i = 0$ so $\mathcal{E} - L\dfrac{di}{dt} = 0$

$\dfrac{di}{dt} = \dfrac{\mathcal{E}}{L} = \dfrac{6.00\ \text{V}}{2.50\ \text{H}} = 2.40\ \text{A/s}$

(b) $\mathcal{E} - iR - L\dfrac{di}{dt} = 0$ (Use this equation rather than Eq. (30.15) since i rather than t is given.)

Thus $\dfrac{di}{dt} = \dfrac{\mathcal{E} - iR}{L} = \dfrac{6.00\ \text{V} - (0.500\ \text{A})(8.00\ \Omega)}{2.50\ \text{H}} = 0.800\ \text{A/s}$

(c) $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) = \left(\dfrac{6.00\ \text{V}}{8.00\ \Omega}\right)(1 - e^{-(8.00\ \Omega/2.50\ \text{H})(0.250\ \text{s})}) = 0.750\ \text{A}(1 - e^{-0.800}) = 0.413\ \text{A}$

(d) Final steady state means $t \to \infty$ and $\dfrac{di}{dt} \to 0$, so $\mathcal{E} - iR = 0$.

$i = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\ \text{V}}{8.00\ \Omega} = 0.750\ \text{A}$

EVALUATE: Our results agree with Figure 30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is di/dt, decreases with t.

30.24. **IDENTIFY:** With S_1 closed and S_2 open, the current builds up to a steady value. Then with S_1 open and S_2 closed, the current decreases exponentially.

SET UP: The decreasing current is $i = I_0 e^{-(R/L)t}$.

EXECUTE: **(a)** $i = I_0 e^{-(R/L)t} = \dfrac{\mathcal{E}}{R} e^{-(R/L)t}$. $e^{-(R/L)t} = \dfrac{iR}{\mathcal{E}} = \dfrac{(0.320\ \text{A})(15.0\ \Omega)}{6.30\ \text{V}} = 0.7619$. $\dfrac{Rt}{L} = -\ln(0.7619)$.

$L = -\dfrac{Rt}{\ln(0.7619)} = -\dfrac{(15.0\ \Omega)(2.00\times10^{-3}\ \text{s})}{\ln(0.7619)} = 0.110\ \text{H}.$

(b) $\dfrac{i}{I_0} = e^{-(R/L)t}$. $e^{-(R/L)t} = 0.0100$. $\dfrac{Rt}{L} = -\ln(0.0100)$.

$$t = -\frac{\ln(0.0100)L}{R} = -\frac{\ln(0.0100)(0.110\text{ H})}{15.0\,\Omega} = 0.0338\text{ s} = 33.8\text{ ms}.$$

EVALUATE: Typical *LR* circuits change rapidly compared to human time scales, so 33.8 ms is not unusual.

30.25. **IDENTIFY:** $i = \mathcal{E}/R(1-e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2}Li^2$.

SET UP: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

EXECUTE: **(a)** $i_{\max} = \mathcal{E}/R$ so $i = i_{\max}/2$ when $(1-e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln\left(\frac{1}{2}\right)$.

$$t = \frac{L\ln 2}{R} = \frac{(\ln 2)(1.25\times10^{-3}\text{ H})}{50.0\,\Omega} = 17.3\,\mu\text{s}$$

(b) $U = \frac{1}{2}U_{\max}$ when $i = i_{\max}/\sqrt{2}$. $1-e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1-1/\sqrt{2} = 0.2929$.

$t = -L\ln(0.2929)/R = 30.7\,\mu\text{s}$.

EVALUATE: $\tau = L/R = 2.50\times10^{-5}\text{ s} = 25.0\,\mu s$. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ.

30.26. **IDENTIFY:** With S_1 closed and S_2 open, $i(t)$ is given by Eq. (30.14). With S_1 open and S_2 closed, $i(t)$ is given by Eq. (30.18).

SET UP: $U = \frac{1}{2}Li^2$. After S_1 has been closed a long time, *i* has reached its final value of $I = \mathcal{E}/R$.

EXECUTE: **(a)** $U = \frac{1}{2}LI^2$ and $I = \sqrt{\dfrac{2U}{L}} = \sqrt{\dfrac{2(0.260\text{ J})}{0.115\text{ H}}} = 2.13\text{ A}$. $\mathcal{E} = IR = (2.13\text{ A})(120\,\Omega) = 256\text{ V}$.

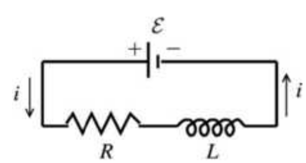
(b) $i = Ie^{-(R/L)t}$ and $U = \frac{1}{2}Li^2 = \frac{1}{2}LI^2e^{-2(R/L)t} = \frac{1}{2}U_0 = \frac{1}{2}\left(\frac{1}{2}LI^2\right)$. $e^{-2(R/L)t} = \frac{1}{2}$, so

$$t = -\frac{L}{2R}\ln\left(\frac{1}{2}\right) = -\frac{0.115\text{ H}}{2(120\,\Omega)}\ln\left(\frac{1}{2}\right) = 3.32\times10^{-4}\text{ s}.$$

EVALUATE: $\tau = L/R = 9.58\times10^{-4}$ s. The time in part (b) is $\tau\ln(2)/2 = 0.347\tau$.

30.27. **IDENTIFY:** Apply the concepts of current decay in an *R*-*L* circuit. Apply the loop rule to the circuit. $i(t)$ is given by Eq. (30.18). The voltage across the resistor depends on *i* and the voltage across the inductor depends on di/dt.

SET UP: The circuit with S_1 closed and S_2 open is sketched in Figure 30.27a.



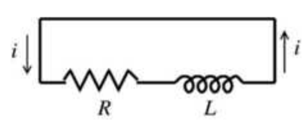
$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Figure 30.27a

Constant current established means $\dfrac{di}{dt} = 0$.

EXECUTE: $i = \dfrac{\mathcal{E}}{R} = \dfrac{60.0\text{ V}}{240\,\Omega} = 0.250\text{ A}$

(a) SET UP: The circuit with S_2 closed and S_1 open is shown in Figure 30.27b.



$i = I_0e^{-(R/L)t}$

At $t = 0, i = I_0 = 0.250\text{ A}$

Figure 30.27b

The inductor prevents an instantaneous change in the current; the current in the inductor just after S_2 is closed and S_1 is opened equals the current in the inductor just before this is done.

(b) EXECUTE: $i = I_0 e^{-(R/L)t} = (0.250\text{ A})e^{-(240\,\Omega/0.160\text{ H})(4.00\times10^{-4}\text{ s})} = (0.250\text{ A})e^{-0.600} = 0.137\text{ A}$

(c) SET UP: See Figure 30.27c.

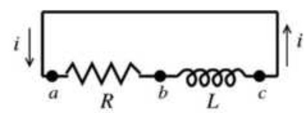


Figure 30.27c

EXECUTE: If we trace around the loop in the direction of the current the potential falls as we travel through the resistor so it must rise as we pass through the inductor: $v_{ab} > 0$ and $v_{bc} < 0$. So point *c* is at a higher potential than point *b*.

$v_{ab} + v_{bc} = 0$ and $v_{bc} = -v_{ab}$

Or, $v_{cb} = v_{ab} = iR = (0.137\text{ A})(240\,\Omega) = 32.9\text{ V}$

(d) $i = I_0 e^{-(R/L)t}$

$i = \frac{1}{2}I_0$ says $\frac{1}{2}I_0 = I_0 e^{-(R/L)t}$ and $\frac{1}{2} = e^{-(R/L)t}$

Taking natural logs of both sides of this equation gives $\ln\left(\frac{1}{2}\right) = -Rt/L$.

$$t = \left(\frac{0.160\text{ H}}{240\,\Omega}\right)\ln 2 = 4.62\times10^{-4}\text{ s}$$

EVALUATE: The current decays, as shown in Figure 30.13 in the textbook. The time constant is $\tau = L/R = 6.67\times10^{-4}$ s. The values of t in the problem are less than one time constant. At any instant the potential drop across the resistor (in the direction of the current) equals the potential rise across the inductor.

30.28. IDENTIFY: Apply Eq. (30.14).

SET UP: $v_{ab} = iR$. $v_{bc} = L\frac{di}{dt}$. The current is increasing, so di/dt is positive.

EXECUTE: (a) At $t = 0$, $i = 0$. $v_{ab} = 0$ and $v_{bc} = 60$ V.

(b) As $t \to \infty$, $i \to \mathcal{E}/R$ and $di/dt \to 0$. $v_{ab} \to 60$ V and $v_{bc} \to 0$.

(c) When $i = 0.150$ A, $v_{ab} = iR = 36.0$ V and $v_{bc} = 60.0\text{ V} - 36.0\text{ V} = 24.0\text{ V}$.

EVALUATE: At all times, $\mathcal{E} = v_{ab} + v_{bc}$, as required by the loop rule.

30.29. IDENTIFY: $i(t)$ is given by Eq. (30.14).

SET UP: The power input from the battery is $\mathcal{E}i$. The rate of dissipation of energy in the resistance is i^2R. The voltage across the inductor has magnitude $L\,di/dt$, so the rate at which energy is being stored in the inductor is $iL\,di/dt$.

EXECUTE: (a) $P = \mathcal{E}i = \mathcal{E}I_0(1 - e^{-(R/L)t}) = \frac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t}) = \frac{(6.00\text{ V})^2}{8.00\,\Omega}(1 - e^{-(8.00\,\Omega/2.50\text{ H})t}).$

$P = (4.50\text{ W})(1 - e^{-(3.20\text{ s}^{-1})t}).$

(b) $P_R = i^2R = \frac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t})^2 = \frac{(6.00\text{ V})^2}{8.00\,\Omega}(1 - e^{-(8.00\,\Omega/2.50\text{ H})t})^2 = (4.50\text{ W})(1 - e^{-(3.20\text{ s}^{-1})t})^2$

(c) $P_L = iL\frac{di}{dt} = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})L\left(\frac{\mathcal{E}}{L}e^{-(R/L)t}\right) = \frac{\mathcal{E}^2}{R}(e^{-(R/L)t} - e^{-2(R/L)t})$

$P_L = (4.50\text{ W})(e^{-(3.20\text{ s}^{-1})t} - e^{-(6.40\text{ s}^{-1})t}).$

EVALUATE: **(d)** Note that if we expand the square in part (b), then parts (b) and (c) add to give part (a), and the total power delivered is dissipated in the resistor and inductor. Conservation of energy requires that this be so.

30.30. **IDENTIFY:** With S_1 closed and S_2 open, the current builds up to a steady value.

SET UP: Applying Kirchhoff's loop rule gives $\mathcal{E} - iR - L\dfrac{di}{dt} = 0.$

EXECUTE: $v_R = \mathcal{E} - L\dfrac{di}{dt} = 18.0\text{ V} - (0.380\text{ H})(7.20\text{ A/s}) = 15.3\text{ V}.$

EVALUATE: The rest of the 18.0 V of the emf is across the inductor.

30.31. **IDENTIFY:** Evaluate $\dfrac{d^2q}{dt^2}$ and insert into Eq. (30.20).

SET UP: Eq. (30.20) is $\dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = 0.$

EXECUTE: $q = Q\cos(\omega t + \phi) \Rightarrow \dfrac{dq}{dt} = -\omega Q\sin(\omega t + \phi) \Rightarrow \dfrac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi).$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = -\omega^2 Q\cos(\omega t + \phi) + \frac{Q}{LC}\cos(\omega t + \phi) = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

EVALUATE: The value of ϕ depends on the initial conditions, the value of q at $t = 0$.

30.32. **IDENTIFY:** An L-C circuit oscillates, with the energy going back and forth between the inductor and capacitor.

(a) SET UP: The frequency is $f = \dfrac{\omega}{2\pi}$ and $\omega = \dfrac{1}{\sqrt{LC}}$, giving $f = \dfrac{1}{2\pi\sqrt{LC}}.$

EXECUTE: $f = \dfrac{1}{2\pi\sqrt{(0.280\times10^{-3}\text{ H})(20.0\times10^{-6}\text{ F})}} = 2.13\times10^3\text{ Hz} = 2.13\text{ kHz}$

(b) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2.$

EXECUTE: $U = \frac{1}{2}(20.0\times10^{-6}\text{ F})(150.0\text{ V})^2 = 0.225\text{ J}$

(c) SET UP: The current in the circuit is $i = -\omega Q\sin\omega t$, and the energy stored in the inductor is $U = \frac{1}{2}Li^2.$

EXECUTE: First find ω and Q. $\omega = 2\pi f = 1.336\times10^4$ rad/s.

$$Q = CV = (20.0\times10^{-6}\text{ F})(150.0\text{ V}) = 3.00\times10^{-3}\text{ C}$$

Now calculate the current:

$$i = -(1.336\times10^4\text{ rad/s})(3.00\times10^{-3}\text{ C})\sin[(1.336\times10^4\text{ rad/s})(1.30\times10^{-3}\text{ s})]$$

Notice that the argument of the sine is in *radians*, so convert it to degrees if necessary. The result is $i = 39.92$ A.

Now find the energy in the inductor: $U = \frac{1}{2}Li^2 = \frac{1}{2}(0.280\times10^{-3}\text{ H})(39.92\text{ A})^2 = 0.223\text{ J}$

EVALUATE: At the end of 1.30 ms, nearly all the energy is now in the inductor, leaving very little in the capacitor.

30.33. **IDENTIFY:** The energy moves back and forth between the inductor and capacitor.

(a) SET UP: The period is $T = \dfrac{1}{f} = \dfrac{1}{\omega/2\pi} = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC}.$

EXECUTE: Solving for L gives

$$L = \frac{T^2}{4\pi^2 C} = \frac{(8.60\times10^{-5}\text{ s})^2}{4\pi^2(7.50\times10^{-9}\text{ C})} = 2.50\times10^{-2}\text{ H} = 25.0\text{ mH}$$

(b) SET UP: The charge on a capacitor is $Q = CV$.

EXECUTE: $Q = CV = (7.50\times10^{-9}\text{ F})(12.0\text{ V}) = 9.00\times10^{-8}\text{ C}$

(c) SET UP: The stored energy is $U = Q^2/2C$.

EXECUTE: $U = \dfrac{(9.00\times10^{-8}\ \text{C})^2}{2(7.50\times10^{-9}\ \text{F})} = 5.40\times10^{-7}\ \text{J}$

(d) SET UP: The maximum current occurs when the capacitor is discharged, so the inductor has all the initial energy. $U_L + U_C = U_{\text{Total}}$. $\frac{1}{2}LI^2 + 0 = U_{\text{Total}}$.

EXECUTE: Solve for the current:

$$I = \sqrt{\frac{2U_{\text{Total}}}{L}} = \sqrt{\frac{2(5.40\times10^{-7}\ \text{J})}{2.50\times10^{-2}\ \text{H}}} = 6.58\times10^{-3}\ \text{A} = 6.58\ \text{mA}$$

EVALUATE: The energy oscillates back and forth forever. However, if there is any resistance in the circuit, no matter how small, all this energy will eventually be dissipated as heat in the resistor.

30.34. IDENTIFY: The circuit is described in Figure 30.14 of the textbook.

SET UP: The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ and the energy stored in the capacitor is $U_C = q^2/2C$. Initially, $U_C = \frac{1}{2}CV^2$, with $V = 22.5$ V. The period of oscillation is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(12.0\times10^{-3}\ \text{H})(18.0\times10^{-6}\ \text{F})} = 2.92$ ms.

EXECUTE: **(a)** Energy conservation says $U_L(\text{max}) = U_C(\text{max})$, and $\frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2}CV^2$.

$i_{\text{max}} = V\sqrt{C/L} = (22.5\ \text{V})\sqrt{\dfrac{18\times10^{-6}\ \text{F}}{12\times10^{-3}\ \text{H}}} = 0.871$ A. The charge on the capacitor is zero because all the energy is in the inductor.

(b) From Figure 30.14 in the textbook, $q = 0$ at $t = T/4 = 0.730$ ms and at $t = 3T/4 = 2.19$ ms.

(c) $q_0 = CV = (18\ \mu\text{F})(22.5\ \text{V}) = 405\ \mu\text{C}$ is the maximum charge on the plates. The graphs are sketched in Figure 30.34. q refers to the charge on one plate and the sign of i indicates the direction of the current.

EVALUATE: If the capacitor is fully charged at $t = 0$ it is fully charged again at $t = T/2$, but with the opposite polarity.

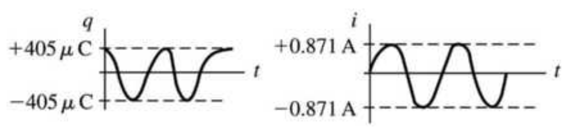
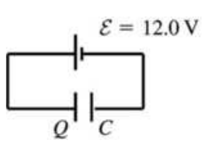


Figure 30.34

30.35. IDENTIFY and SET UP: The angular frequency is given by Eq. (30.22). $q(t)$ and $i(t)$ are given by Eqs. (30.21) and (30.23). The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.50\ \text{H})(6.00\times10^{-5}\ \text{F})}} = 105.4\ \text{rad/s}$, which rounds to $105\ \text{rad/s}$. The period is given by $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{105.4\ \text{rad/s}} = 0.0596$ s.

(b) The circuit containing the battery and capacitor is sketched in Figure 30.35.



$$\mathcal{E} - \frac{Q}{C} = 0$$

$$Q = \mathcal{E}C = (12.0\ \text{V})(6.00\times10^{-5}\ \text{F}) = 7.20\times10^{-4}\ \text{C}$$

Figure 30.35

(c) $U=\frac{1}{2}CV^2=\frac{1}{2}(6.00\times10^{-5}\text{ F})(12.0\text{ V})^2=4.32\times10^{-3}\text{ J}$

(d) $q=Q\cos(\omega t+\phi)$ (Eq. 30.21)

$q=Q$ at $t=0$ so $\phi=0$

$q=Q\cos\omega t=(7.20\times10^{-4}\text{ C})\cos([105.4\text{ rad/s}][0.0230\text{ s}])=-5.42\times10^{-4}\text{ C}$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) $i=-\omega Q\sin(\omega t+\phi)$ (Eq. 30.23)

$i=-(105\text{ rad/s})(7.20\times10^{-4}\text{ C})\sin([105.4\text{ rad/s}][0.0230\text{ s}])=-0.050\text{ A}$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\frac{1}{2}Li^2+\frac{q^2}{2C}=\frac{Q^2}{2C}$ gives $i=\pm\sqrt{\frac{1}{LC}}\sqrt{Q^2-q^2}$ (Eq. 30.26)

$i=\pm(105\text{ rad/s})\sqrt{(7.20\times10^{-4}\text{ C})^2-(-5.42\times10^{-4}\text{ C})^2}=\pm0.050\text{ A},$ which checks.

(f) $U_C=\frac{q^2}{2C}=\frac{(-5.42\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})}=2.45\times10^{-3}\text{ J}$

$U_L=\frac{1}{2}Li^2=\frac{1}{2}(1.50\text{ H})(0.050\text{ A})^2=1.87\times10^{-3}\text{ J}$

EVALUATE: Note that $U_C+U_L=2.45\times10^{-3}\text{ J}+1.87\times10^{-3}\text{ J}=4.32\times10^{-3}\text{ J}.$

This agrees with the total energy initially stored in the capacitor,

$U=\frac{Q^2}{2C}=\frac{(7.20\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})}=4.32\times10^{-3}\text{ J}.$

Energy is conserved. At some times there is energy stored in both the capacitor and the inductor. When $i=0$ all the energy is stored in the capacitor and when $q=0$ all the energy is stored in the inductor. But at all times the total energy stored is the same.

30.36. **IDENTIFY:** $\omega=\frac{1}{\sqrt{LC}}=2\pi f$

SET UP: ω is the angular frequency in rad/s and f is the corresponding frequency in Hz.

EXECUTE: **(a)** $L=\frac{1}{4\pi^2f^2C}=\frac{1}{4\pi^2(1.6\times10^6\text{ Hz})^2(4.18\times10^{-12}\text{ F})}=2.37\times10^{-3}\text{ H}.$

(b) The maximum capacitance corresponds to the minimum frequency.

$C_{\text{max}}=\frac{1}{4\pi^2f_{\text{min}}^2L}=\frac{1}{4\pi^2(5.40\times10^5\text{ Hz})^2(2.37\times10^{-3}\text{ H})}=3.67\times10^{-11}\text{ F}=36.7\text{ pF}$

EVALUATE: To vary f by a factor of three (approximately the range in this problem), C must be varied by a factor of nine.

30.37. **IDENTIFY:** Apply energy conservation and Eqs. (30.22) and (30.23).

SET UP: If I is the maximum current, $\frac{1}{2}LI^2=\frac{Q^2}{2C}$. For the inductor, $U_L=\frac{1}{2}Li^2$.

EXECUTE: **(a)** $\frac{1}{2}LI^2=\frac{Q^2}{2C}$ gives $Q=I\sqrt{LC}=(0.750\text{ A})\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})}=7.50\times10^{-6}\text{ C}.$

(b) $\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})}}=1.00\times10^5\text{ rad/s}.$ $f=\frac{\omega}{2\pi}=1.59\times10^4\text{ Hz}.$

(c) $q=Q$ at $t=0$ means $\phi=0$. $i=-\omega Q\sin(\omega t)$, so

$i=-(1.00\times10^5\text{ rad/s})(7.50\times10^{-6}\text{ C})\sin([1.00\times10^5\text{ rad/s}][2.50\times10^{-3}\text{ s}])=0.7279\text{ A}.$

$U_L=\frac{1}{2}Li^2=\frac{1}{2}(0.0800\text{ H})(0.7279\text{ A})^2=0.0212\text{ J}.$

EVALUATE: The total energy of the system is $\frac{1}{2}LI^2 = 0.0225$ J. At $t = 2.50$ ms, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.38. **IDENTIFY:** Apply Eq. (30.25).

SET UP: $q = Q$ when $i = 0$. $i = i_{max}$ when $q = 0$. $1/\sqrt{LC} = 1917\ \text{s}^{-1}$.

EXECUTE: **(a)** $\frac{1}{2}Li_{max}^2 = \dfrac{Q^2}{2C}$.

$Q = i_{max}\sqrt{LC} = (0.850\times10^{-3}\ \text{A})\sqrt{(0.0850\ \text{H})(3.20\times10^{-6}\ \text{F})} = 4.43\times10^{-7}\ \text{C}$

(b) $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43\times10^{-7}\ \text{C})^2 - \left(\dfrac{5.00\times10^{-4}\ \text{A}}{1917\ \text{s}^{-1}}\right)^2} = 3.58\times10^{-7}\ \text{C}$.

EVALUATE: The value of q calculated in part (b) is less than the maximum value Q calculated in part (a).

30.39. **IDENTIFY:** Evaluate Eq. (30.29).

SET UP: The angular frequency of the circuit is ω'.

EXECUTE: **(a)** When $R = 0$, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\ \text{H})(2.50\times10^{-5}\ \text{F})}} = 298\ \text{rad/s}$.

(b) We want $\dfrac{\omega'}{\omega_0} = 0.95$, so $\dfrac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \dfrac{R^2C}{4L} = (0.95)^2$. This gives

$R = \sqrt{\dfrac{4L}{C}(1-(0.95)^2)} = \sqrt{\dfrac{4(0.450\ \text{H})(0.0975)}{(2.50\times10^{-5}\ \text{F})}} = 83.8\ \Omega$.

EVALUATE: When R increases, the angular frequency decreases and approaches zero as $R \to 2\sqrt{L/C}$.

30.40. **IDENTIFY:** The presence of resistance in an L-R-C circuit affects the frequency of oscillation and causes the amplitude of the oscillations to decrease over time.

(a) SET UP: The frequency of damped oscillations is $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$.

EXECUTE: $\omega' = \sqrt{\dfrac{1}{(22\times10^{-3}\ \text{H})(15.0\times10^{-9}\ \text{F})} - \dfrac{(75.0\ \Omega)^2}{4(22\times10^{-3}\ \text{H})^2}} = 5.5\times10^4\ \text{rad/s}$

The frequency f is $f = \dfrac{\omega}{2\pi} = \dfrac{5.50\times10^4\ \text{rad/s}}{2\pi} = 8.76\times10^3\ \text{Hz} = 8.76\ \text{kHz}$.

(b) SET UP: The amplitude decreases as $A(t) = A_0\ e^{-(R/2L)t}$.

Execute**:** Solving for t and putting in the numbers gives:

$$t = \frac{-2L\ln(A/A_0)}{R} = \frac{-2(22.0\times10^{-3}\ \text{H})\ln(0.100)}{75.0\ \Omega} = 1.35\times10^{-3}\ \text{s} = 1.35\ \text{ms}$$

(c) SET UP: At critical damping, $R = \sqrt{4L/C}$.

EXECUTE: $R = \sqrt{\dfrac{4(22.0\times10^{-3}\ \text{H})}{15.0\times10^{-9}\ \text{F}}} = 2420\ \Omega$

EVALUATE: The frequency with damping is almost the same as the resonance frequency of this circuit $(1/\sqrt{LC})$, which is plausible because the 75-Ω resistance is considerably less than the $2420\ \Omega$ required for critical damping.

30.41. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: Make the substitutions $x \to q, m \to L, b \to R, k \to \dfrac{1}{C}$.

EXECUTE: **(a)** Eq. (14.41): $\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{kx}{m} = 0$. This becomes $\dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0$, which is Eq. (30.27).

(b) Eq. (14.43): $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$. This becomes $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$, which is Eq. (30.29).

(c) Eq. (14.42): $x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$. This becomes $q = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$, which is Eq. (30.28).

EVALUATE: Equations for the *L-R-C* circuit and for a damped harmonic oscillator have the same form.

30.42. **IDENTIFY:** For part (a), evaluate the derivatives as specified in the problem. For part (b) set $q = Q$ in Eq. (30.28) and set $dq/dt = 0$ in the expression for dq/dt.

SET UP: In terms of ω', Eq. (30.28) is $q(t) = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$.

EXECUTE: **(a)** $q = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$. $\dfrac{dq}{dt} = -A\dfrac{R}{2L}e^{-(R/2L)t}\cos(\omega' t + \phi) - \omega' Ae^{-(R/2L)t}\sin(\omega' t + \phi)$.

$$\frac{d^2q}{dt^2} = A\left(\frac{R}{2L}\right)^2 e^{-(R/2L)t}\cos(\omega' t + \phi) + 2\omega' A\frac{R}{2L}e^{-(R/2L)t}\sin(\omega' t + \phi) - \omega'^2 Ae^{-(R/2L)t}\cos(\omega' t + \phi).$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = q\left(\left(\frac{R}{2L}\right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC}\right) = 0, \text{ so } \omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2}.$$

(b) At $t = 0, q = Q, i = \dfrac{dq}{dt} = 0$, so $q = A\cos\phi = Q$ and $\dfrac{dq}{dt} = -\dfrac{R}{2L}A\cos\phi - \omega' A\sin\phi = 0$. This gives

$$A = \frac{Q}{\cos\phi} \text{ and } \tan\phi = -\frac{R}{2L\omega'} = -\frac{R}{2L\sqrt{1/LC - R^2/4L^2}}.$$

EVALUATE: If $R = 0$, then $A = Q$ and $\phi = 0$.

30.43. **IDENTIFY** and **SET UP:** The emf $\mathcal{E}_2$ in solenoid 2 produced by changing current i_1 in solenoid 1 is given by $\mathcal{E}_2 = M\left|\dfrac{di_1}{dt}\right|$. The mutual inductance of two solenoids is derived in Example 30.1. For the two solenoids in this problem $M = \dfrac{\mu_0 AN_1N_2}{l}$, where *A* is the cross-sectional area of the inner solenoid and *l* is the length of the outer solenoid. Let the outer solenoid be solenoid 1.

EXECUTE: **(a)** $M = \dfrac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})\pi(6.00\times 10^{-4}\text{ m})^2(6750)(15)}{0.500\text{ m}} = 2.88\times 10^{-7}\text{ H} = 0.288\ \mu\text{H}.$

(b) $\mathcal{E}_2 = M\left|\dfrac{di_1}{dt}\right| = (2.88\times 10^{-7}\text{ H})(49.2\text{ A/s}) = 1.42\times 10^{-5}\text{ V}.$

EVALUATE: If current in the inner solenoid changed at 49.2 A/s, the emf induced in the outer solenoid would be 1.42×10^{-5} V.

30.44. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$ and $Li = N\Phi_B$.

SET UP: Φ_B is the flux through one turn.

EXECUTE: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -(4.80\times 10^{-3}\text{ H})\dfrac{d}{dt}\{(0.680\text{ A})\cos[\pi t/(0.0250\text{ s})]\}.$

$\mathcal{E} = (4.80\times 10^{-3}\text{ H})(0.680\text{ A})\dfrac{\pi}{0.0250\text{ s}}\sin(\pi t/[0.0250\text{ s}])$. Therefore,

$$\mathcal{E}_{\text{max}} = (4.80\times 10^{-3}\text{ H})(0.680\text{ A})\frac{\pi}{0.0250\text{ s}} = 0.410\text{ V}.$$

(b) $\Phi_{B\,\text{max}} = \dfrac{Li_{\text{max}}}{N} = \dfrac{(4.80\times 10^{-3}\text{ H})(0.680\text{ A})}{400} = 8.16\times 10^{-6}\text{ Wb}.$

(c) $\mathcal{E}(t) = -L\dfrac{di}{dt} = (4.80\times10^{-3}\,\text{H})(0.680\,\text{A})(\pi/0.0250\,\text{s})\sin(\pi t/0.0250\,\text{s})$. $\mathcal{E}(t) = (0.410\,\text{V})\sin[(125.6\,\text{s}^{-1})t]$.

Therefore, at $t = 0.0180$ s, $\mathcal{E}(0.0180\,\text{s}) = (0.410\,\text{V})\sin[(125.6\,\text{s}^{-1})(0.0180\,\text{s})] = 0.316$ V. The magnitude of the induced emf is 0.316 V.

EVALUATE: The maximum emf is when $i = 0$ and at this instant $\Phi_B = 0$.

30.45. **IDENTIFY:** $\mathcal{E} = -L\dfrac{di}{dt}$.

SET UP: During an interval in which the graph of *i* versus *t* is a straight line, di/dt is constant and equal to the slope of that line.

EXECUTE: **(a)** The pattern on the oscilloscope is sketched in Figure 30.45.

EVALUATE: **(b)** Since the voltage is determined by the derivative of the current, the *V* versus *t* graph is indeed proportional to the derivative of the current graph

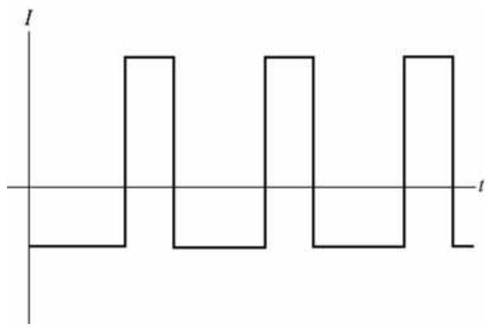


Figure 30.45

30.46. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$.

SET UP: $\dfrac{d}{dt}\cos(\omega t) = -\omega\sin(\omega t)$

EXECUTE: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}((0.124\,\text{A})\cos[(240\,\pi/\text{s})t]$.

$$\mathcal{E} = +(0.250\,\text{H})(0.124\,\text{A})(240\,\pi/\text{s})\sin((240\pi/\text{s})t) = +(23.4\,\text{V})\sin((240\pi/\text{s})t).$$

The graphs are given in Figure 30.46.

(b) $\mathcal{E}_{\text{max}} = 23.4$ V; $i = 0$, since the emf and current are $90°$ out of phase.

(c) $i_{\text{max}} = 0.124$ A; $\mathcal{E} = 0$, since the emf and current are $90°$ out of phase.

EVALUATE: The induced emf depends on the rate at which the current is changing.

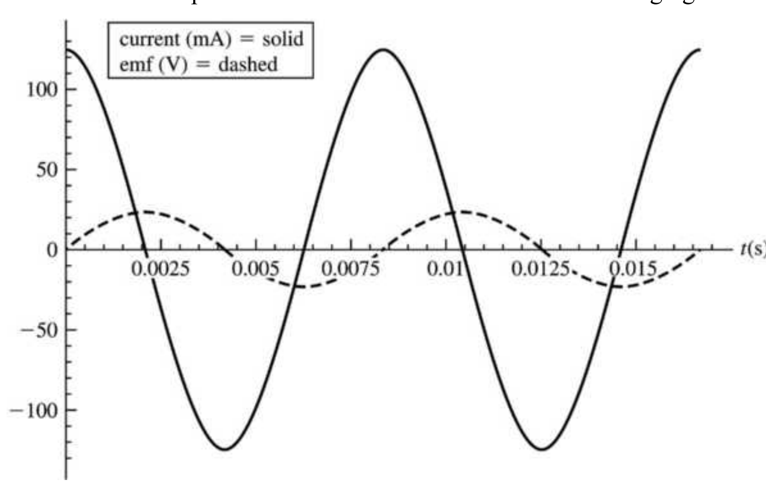


Figure 30.46

30.47. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\text{ m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times 10^4\text{ J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$. $K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times 10^4\text{ m/s}$.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.48. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}$.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} l dr$.

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 i l}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i l}{2\pi}\ln(b/a)$.

(d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a)$.

(e) $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}l\dfrac{\mu_0}{2\pi}\ln(b/a)i^2 = \dfrac{\mu_0 l i^2}{4\pi}\ln(b/a)$.

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.49. **(a) IDENTIFY** and **SET UP:** An end view is shown in Figure 30.49.

Apply Ampere's law to a circular path of radius r.
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$

Figure 30.49

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$

$I_{\text{encl}} = i$, the current in the inner conductor

Thus $B(2\pi r) = \mu_0 i$ and $B = \dfrac{\mu_0 i}{2\pi r}$.

(b) IDENTIFY and **SET UP:** Follow the procedure specified in the problem.

EXECUTE: $u = \dfrac{B^2}{2\mu_0}$

$dU = u\,dV$, where $dV = 2\pi r l dr$

$$dU = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2 (2\pi r l)dr = \frac{\mu_0 i^2 l}{4\pi r}dr$$

(c) $U = \int dU = \dfrac{\mu_0 i^2 l}{4\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i^2 l}{4\pi}[\ln r]_a^b$

$$U = \frac{\mu_0 i^2 l}{4\pi}(\ln b - \ln a) = \frac{\mu_0 i^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

(d) Eq. (30.9): $U = \frac{1}{2}Li^2$

Part (c): $U = \dfrac{\mu_0 i^2 l}{4\pi}\ln\left(\dfrac{b}{a}\right)$

$\frac{1}{2}Li^2 = \dfrac{\mu_0 i^2 l}{4\pi}\ln\left(\dfrac{b}{a}\right)$

$L = \dfrac{\mu_0 l}{2\pi}\ln\left(\dfrac{b}{a}\right).$

EVALUATE: The value of L we obtain from these energy considerations agrees with L calculated in part (d) of Problem 30.48 by considering flux and Eq. (30.6).

30.50. **IDENTIFY:** Apply $L = \dfrac{N\Phi_B}{i}$ to each solenoid, as in Example 30.3. Use $M = \dfrac{N_2\Phi_{B2}}{i_1}$ to calculate the mutual inductance M.

SET UP: The magnetic field produced by solenoid 1 is confined to the space within its windings and is equal to $B_1 = \dfrac{\mu_0 N_1 i_1}{2\pi r}$.

EXECUTE: **(a)** $L_1 = \dfrac{N_1\Phi_{B_1}}{i_1} = \dfrac{N_1 A}{i_1}\left(\dfrac{\mu_0 N_1 i_1}{2\pi r}\right) = \dfrac{\mu_0 N_1^2 A}{2\pi r}$, $L_2 = \dfrac{N_2\Phi_{B_2}}{i_2} = \dfrac{N_2 A}{i_2}\left(\dfrac{\mu_0 N_2 i_2}{2\pi r}\right) = \dfrac{\mu_0 N_2^2 A}{2\pi r}$.

(b) $M = \dfrac{N_2 A B_1}{i_1} = \dfrac{\mu_0 N_1 N_2 A}{2\pi r}$. $M^2 = \left(\dfrac{\mu_0 N_1 N_2 A}{2\pi r}\right)^2 = \dfrac{\mu_0 N_1^2 A}{2\pi r}\dfrac{\mu_0 N_2^2 A}{2\pi r} = L_1 L_2$.

EVALUATE: If the two solenoids are identical, so that $N_1 = N_2$, then $M = L$.

30.51. **IDENTIFY:** $U = \frac{1}{2}LI^2$. The self-inductance of a solenoid is found in Exercise 30.15 to be $L = \dfrac{\mu_0 A N^2}{l}$.

SET UP: The length l of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: **(a)** $L = \dfrac{2U}{I^2} = \dfrac{2(10.0\text{ J})}{(2.00\text{ A})^2} = 5.00\text{ H}.$

(b) $L = \dfrac{\mu_0 A N^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A\alpha^2 l$. For this coil $\alpha = 10\text{ coils/mm} = 10\times10^3\text{ coils/m}$. Solving for l gives

$l = \dfrac{L}{\mu_0 A\alpha^2} = \dfrac{5.00\text{ H}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\pi(0.0200\text{ m})^2(10\times10^3\text{ coils/m})^2} = 31.7\text{ m}$. This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (31.7\text{ m})(10\times10^3\text{ coils/m}) = 3.17\times10^5$ turns. The length of wire in the solenoid is the circumference C of one turn times the number of turns.

$C = \pi d = \pi(4.00\times10^{-2}\text{ m}) = 0.126\text{ m}$. The length of wire is $(0.126\text{ m})(3.17\times10^5) = 4.0\times10^4\text{ m} = 40\text{ km}$.

This length of wire will have a large resistance and I^2R electrical energy loses will be very large.

30.52. **IDENTIFY:** This is an R-L circuit and $i(t)$ is given by Eq. (30.14).

SET UP: When $t \to \infty$, $i \to i_\text{f} = V/R$.

EXECUTE: **(a)** $R = \dfrac{V}{i_\text{f}} = \dfrac{12.0\text{ V}}{6.45\times10^{-3}\text{ A}} = 1860\ \Omega.$

(b) $i = i_\text{f}(1 - e^{-(R/L)t})$ so $\dfrac{Rt}{L} = -\ln(1 - i/i_\text{f})$ and $L = \dfrac{-Rt}{\ln(1 - i/i_\text{f})} = \dfrac{-(1860\ \Omega)(9.40\times10^{-4}\text{s})}{\ln(1-(4.86/6.45))} = 1.25\text{ H}.$

EVALUATE: The current after a long time depends only on R and is independent of L. The value of R/L determines how rapidly the final value of i is reached.

30.53. **IDENTIFY** and **SET UP:** Follow the procedure specified in the problem. $L = 2.50$ H, $R = 8.00\ \Omega$, $\mathcal{E} = 6.00$ V. $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$, $\tau = L/R$

EXECUTE: **(a)** Eq. (30.9): $U_L = \frac{1}{2}Li^2$

$t = \tau$ so $i = (\mathcal{E}/R)(1 - e^{-1}) = (6.00\text{ V}/8.00\ \Omega)(1 - e^{-1}) = 0.474$ A

Then $U_L = \frac{1}{2}Li^2 = \frac{1}{2}(2.50\text{ H})(0.474\ A)^2 = 0.281$ J

Exercise 30.29 (c): $P_L = \dfrac{dU_L}{dt} = Li\dfrac{di}{dt}$

$$i = \left(\frac{\mathcal{E}}{R}\right)(1 - e^{-t/\tau});\ \frac{di}{dt} = \left(\frac{\mathcal{E}}{L}\right)e^{-(R/L)t} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

$$P_L = L\left(\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right)\left(\frac{\mathcal{E}}{L}e^{-t/\tau}\right) = \frac{\mathcal{E}^2}{R}(e^{-t/\tau} - e^{-2t/\tau})$$

$$U_L = \int_0^\tau P_L dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (e^{-t/\tau} - e^{-2t/\tau})dt = \frac{\mathcal{E}^2}{R}\left[-\tau e^{-t/\tau} + \frac{\tau}{2}e^{-2t/\tau}\right]_0^\tau$$

$$U_L = -\frac{\mathcal{E}^2}{R}\tau\left[e^{-t/\tau} - \tfrac{1}{2}e^{-2t/\tau}\right]_0^\tau = \frac{\mathcal{E}^2}{R}\tau\left[1 - \tfrac{1}{2} - e^{-1} + \tfrac{1}{2}e^{-2}\right]$$

$$U_L = \left(\frac{\mathcal{E}^2}{2R}\right)\left(\frac{L}{R}\right)(1 - 2e^{-1} + e^{-2}) = \tfrac{1}{2}\left(\frac{\mathcal{E}}{R}\right)^2 L(1 - 2e^{-1} + e^{-2})$$

$$U_L = \tfrac{1}{2}\left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2(2.50\text{ H})(0.3996) = 0.281\text{ J, which checks.}$$

(b) Exercise 30.29(a): The rate at which the battery supplies energy is

$$P_\mathcal{E} = \mathcal{E}i = \mathcal{E}\left(\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right) = \frac{\mathcal{E}^2}{R}(1 - e^{-t/\tau})$$

$$U_\mathcal{E} = \int_0^\tau P_\mathcal{E} dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (1 - e^{-t/\tau})dt = \frac{\mathcal{E}^2}{R}[t + \tau e^{-t/\tau}]_0^\tau = \left(\frac{\mathcal{E}^2}{R}\right)(\tau + \tau e^{-1} - \tau)$$

$$U_\mathcal{E} = \left(\frac{\mathcal{E}^2}{R}\right)\tau e^{-1} = \left(\frac{\mathcal{E}^2}{R}\right)\left(\frac{L}{R}\right)e^{-1} = \left(\frac{\mathcal{E}}{R}\right)^2 Le^{-1}$$

$$U_\mathcal{E} = \left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2(2.50\text{ H})(0.3679) = 0.517\text{ J}$$

(c) $P_R = i^2R = \left(\dfrac{\mathcal{E}^2}{R}\right)(1 - e^{-t/\tau})^2 = \dfrac{\mathcal{E}^2}{R}(1 - 2e^{-t/\tau} + e^{-2t/\tau})$

$$U_R = \int_0^\tau P_R dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (1 - 2e^{-t/\tau} + e^{-2t/\tau})dt = \frac{\mathcal{E}^2}{R}\left[t + 2\tau e^{-t/\tau} - \frac{\tau}{2}e^{-2t/\tau}\right]_0^\tau$$

$$U_R = \frac{\mathcal{E}^2}{R}\left[\tau + 2\tau e^{-1} - \frac{\tau}{2}e^{-2} - 2\tau + \frac{\tau}{2}\right] = \frac{\mathcal{E}^2}{R}\left[-\frac{\tau}{2} + 2\tau e^{-1} - \frac{\tau}{2}e^{-2}\right]$$

$$U_R = \left(\frac{\mathcal{E}^2}{2R}\right)\left(\frac{L}{R}\right)[-1 + 4e^{-1} - e^{-2}]$$

$$U_R = \left(\frac{\mathcal{E}}{R}\right)^2\left(\tfrac{1}{2}L\right)[-1 + 4e^{-1} - e^{-2}] = \left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2\tfrac{1}{2}(2.50\text{ H})(0.3362) = 0.236\text{ J}$$

(d) EVALUATE: $U_\mathcal{E} = U_R + U_L$. (0.517 J = 0.236 J + 0.281 J)

The energy supplied by the battery equals the sum of the energy stored in the magnetic field of the inductor and the energy dissipated in the resistance of the inductor.

30.54. **IDENTIFY:** This is a decaying R-L circuit with $I_0 = \mathcal{E}/R.\ i(t) = I_0 e^{-(R/L)t}.$

SET UP: $\mathcal{E} = 60.0$ V, $R = 240\ \Omega$ and $L = 0.160$ H. The rate at which energy stored in the inductor is decreasing is $iLdi/dt$.

EXECUTE: **(a)** $U = \frac{1}{2}LI_0^2 = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}(0.160\text{ H})\left(\frac{60\text{ V}}{240\ \Omega}\right)^2 = 5.00\times10^{-3}$ J.

(b) $i = \frac{\mathcal{E}}{R}e^{-(R/L)t} \Rightarrow \frac{di}{dt} = -\frac{R}{L}i \Rightarrow \frac{dU_L}{dt} = iL\frac{di}{dt} = -Ri^2 = \frac{\mathcal{E}^2}{R}e^{-2(R/L)t}.$

$\frac{dU_L}{dt} = -\frac{(60\text{ V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})} = -4.52$ W.

(c) In the resistor, $P_R = \frac{dU_R}{dt} = i^2R = \frac{\mathcal{E}^2}{R}e^{-2(R/L)t} = \frac{(60\text{ V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})} = 4.52$ W.

(d) $P_R(t) = i^2R = \frac{\mathcal{E}^2}{R}e^{-2(R/L)t}$. $U_R = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2(R/L)t} = \frac{\mathcal{E}^2}{R}\frac{L}{2R} = \frac{(60\text{ V})^2(0.160\text{ H})}{2(240\ \Omega)^2} = 5.00\times10^{-3}$ J, which is the same as part (a).

EVALUATE: During the decay of the current all the electrical energy originally stored in the inductor is dissipated in the resistor.

30.55. **IDENTIFY** and **SET UP:** Follow the procedure specified in the problem. $\frac{1}{2}Li^2$ is the energy stored in the inductor and $q^2/2C$ is the energy stored in the capacitor. The equation is $-iR - L\frac{di}{dt} - \frac{q}{C} = 0.$

EXECUTE: Multiplying by $-i$ gives $i^2R + Li\frac{di}{dt} + \frac{qi}{C} = 0.$

$\frac{d}{dt}U_L = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = \frac{1}{2}L\frac{d}{dt}(i^2) = \frac{1}{2}L\left(2i\frac{di}{dt}\right) = Li\frac{di}{dt}$, the second term.

$\frac{d}{dt}U_C = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \frac{1}{2C}\frac{d}{dt}(q^2) = \frac{1}{2C}(2q)\frac{dq}{dt} = \frac{qi}{C}$, the third term. $i^2R = P_R$, the rate at which electrical energy is dissipated in the resistance. $\frac{d}{dt}U_L = P_L$, the rate at which the amount of energy stored in the inductor is changing. $\frac{d}{dt}U_C = P_C$, the rate at which the amount of energy stored in the capacitor is changing.

EVALUATE: The equation says that $P_R + P_L + P_C = 0$; the net rate of change of energy in the circuit is zero. Note that at any given time one of P_C or P_L is negative. If the current and U_L are increasing the charge on the capacitor and U_C are decreasing, and vice versa.

30.56. **IDENTIFY:** The energy stored in a capacitor is $U_C = \frac{1}{2}Cv^2$. The energy stored in an inductor is $U_L = \frac{1}{2}Li^2$. Energy conservation requires that the total stored energy be constant.

SET UP: The current is a maximum when the charge on the capacitor is zero and the energy stored in the capacitor is zero.

EXECUTE: **(a)** Initially $v = 16.0$ V and $i = 0$. $U_L = 0$ and $U_C = \frac{1}{2}Cv^2 = \frac{1}{2}(7.00\times10^{-6}\text{ F})(16.0\text{ V})^2 = 8.96\times10^{-4}$ J. The total energy stored is 0.896 mJ.

(b) The current is maximum when $q = 0$ and $U_C = 0$. $U_C + U_L = 8.96\times10^{-4}$ J so $U_L = 8.96\times10^{-4}$ J.

$\frac{1}{2}Li_{\text{max}}^2 = 8.96\times10^{-4}$ J and $i_{\text{max}} = \sqrt{\frac{2(8.96\times10^{-4}\text{ J})}{3.75\times10^{-3}\text{ H}}} = 0.691$ A.

EVALUATE: The maximum charge on the capacitor is $Q = CV = 112\ \mu C$.

30.57. **IDENTIFY and SET UP:** Use $U_C = \frac{1}{2}CV_C^2$ (energy stored in a capacitor) to solve for C. Then use Eq. (30.22) and $\omega = 2\pi f$ to solve for the L that gives the desired current oscillation frequency.

EXECUTE: $V_C = 12.0$ V; $U_C = \frac{1}{2}CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160\text{ J})/(12.0\text{ V})^2 = 222\ \mu\text{F}$

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ so } L = \frac{1}{(2\pi f)^2 C}$$

$f = 3500$ Hz gives $L = 9.31\ \mu\text{H}$

EVALUATE: f is in Hz and ω is in rad/s; we must be careful not to confuse the two.

30.58. **IDENTIFY:** Apply energy conservation to the circuit.

SET UP: For a capacitor $V = q/C$ and $U = q^2/2C$. For an inductor $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $V_{\text{max}} = \dfrac{Q}{C} = \dfrac{6.00\times10^{-6}\text{ C}}{2.50\times10^{-4}\text{ F}} = 0.0240\text{ V}.$

(b) $\dfrac{1}{2}Li_{\text{max}}^2 = \dfrac{Q^2}{2C}$, so $i_{\text{max}} = \dfrac{Q}{\sqrt{LC}} = \dfrac{6.00\times10^{-6}\text{ C}}{\sqrt{(0.0600\text{ H})(2.50\times10^{-4}\text{ F})}} = 1.55\times10^{-3}\text{ A}$

(c) $U_{\text{max}} = \dfrac{1}{2}Li_{\text{max}}^2 = \dfrac{1}{2}(0.0600\text{ H})(1.55\times10^{-3}\text{A})^2 = 7.21\times10^{-8}\text{ J}.$

(d) If $i = \dfrac{1}{2}i_{\text{max}}$ then $U_L = \dfrac{1}{4}U_{\text{max}} = 1.80\times10^{-8}$ J and $U_C = \dfrac{3}{4}U_{\text{max}} = \dfrac{\left(\sqrt{(3/4)}Q\right)^2}{2C} = \dfrac{q^2}{2C}$. This gives $q = \sqrt{\dfrac{3}{4}}Q = 5.20\times10^{-6}\text{ C}.$

EVALUATE: $U_{\text{max}} = \dfrac{1}{2}Li^2 + \dfrac{1}{2}\dfrac{q^2}{C}$ for all times.

30.59. **IDENTIFY:** The initial energy stored in the capacitor is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\dfrac{q}{C} = L\left|\dfrac{di}{dt}\right|$. The capacitor energy is $U_C = \dfrac{q^2}{2C}$ and the inductor energy is $U_L = \dfrac{1}{2}Li^2$.

EXECUTE: $\dfrac{q^2}{2C} + \dfrac{1}{2}Li^2 = \dfrac{Q_{\text{max}}^2}{2C}$. $Q_{\text{max}} = (84.0\times10^{-9}\text{ F})(12.0\text{ V}) = 1.008\times10^{-6}\text{ C}$

$$\frac{1}{2}Li^2 = \frac{1}{2C}(Q_{\text{max}}^2 - q^2) = \frac{1}{2(84.0\times10^{-9}\text{ F})}((1.008\times10^{-6}\text{ C})^2 - (0.650\times10^{-6}\text{ C})^2) = 3.533\times10^{-6}\text{ J}.$$

$$i = \sqrt{\frac{2(3.533\times10^{-6}\text{ J})}{0.0420\text{ H}}} = 0.0130\text{ A} = 13.0\text{ mA}.$$

$$\left|\frac{di}{dt}\right| = \frac{q}{LC} = \frac{0.650\times10^{-6}\text{ C}}{(0.0420\text{ H})(84.0\times10^{-9}\text{ F})} = 184\text{ A/s}.$$

EVALUATE: The current is only 13 mA but is changing at a rate of 184 A/s. However, it only changes at that rate for a tiny fraction of a second.

30.60. **IDENTIFY:** The total energy is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\dfrac{q}{C} = L\left|\dfrac{di}{dt}\right|$. The capacitor energy is $U_C = \dfrac{q^2}{2C}$ and the inductor energy is $U_L = \dfrac{1}{2}Li^2$.

EXECUTE: The total energy is $\frac{q^2}{2C}+\frac{1}{2}Li^2=\frac{Q_{max}^2}{2C}=\frac{1}{2}CV_{max}^2.$

$$q=LC\left|\frac{di}{dt}\right|=(0.330\text{ H})(5.90\times10^{-4}\text{ F})(89.0\text{ A/s})=1.733\times10^{-2}\text{ C}.$$

$$\frac{1}{2}CV_{max}^2=\frac{(1.733\times10^{-2}\text{ C})^2}{2(5.90\times10^{-4}\text{ F})}+\frac{1}{2}(0.330\text{ H})(2.50\text{ A})^2=1.286\text{ J}.$$

$$V_{max}=\sqrt{\frac{2(1.286\text{ J})}{5.90\times10^{-4}\text{ F})}}=66.0\text{ V}.$$

EVALUATE: By energy conservation, the maximum potential across the inductor will also be 66.0 V, but that will occur only at the instants when the capacitor is uncharged.

30.61. **IDENTIFY:** The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: First combine the inductors.

EXECUTE: **(a)** Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. *A* reads zero and *V* reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.61a. $I=(20.0\text{ V})/(75.0\;\Omega)=0.267\text{ A}$. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Combine the inductor network into its equivalent, as shown in Figure 30.61b. $R=75.0\;\Omega$ is the equivalent resistance. Eq. (30.14) says $i=(\mathcal{E}/R)(1-e^{-t/\tau})$ with $\tau=L/R=(10.8\text{ mH})/(75.0\;\Omega)=0.144\text{ ms}$. $\mathcal{E}=20.0\text{ V}$, $R=75.0\;\Omega$, $t=0.115\text{ ms}$ so $i=0.147\text{ A}$. $V_R=iR=(0.147\text{ A})(75.0\;\Omega)=11.0\text{ V}$. $20.0\text{ V}-V_R-V_L=0$ and $V_L=20.0\text{ V}-V_R=9.0\text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A . The voltage across the inductor network drops from 20.0 V to zero.

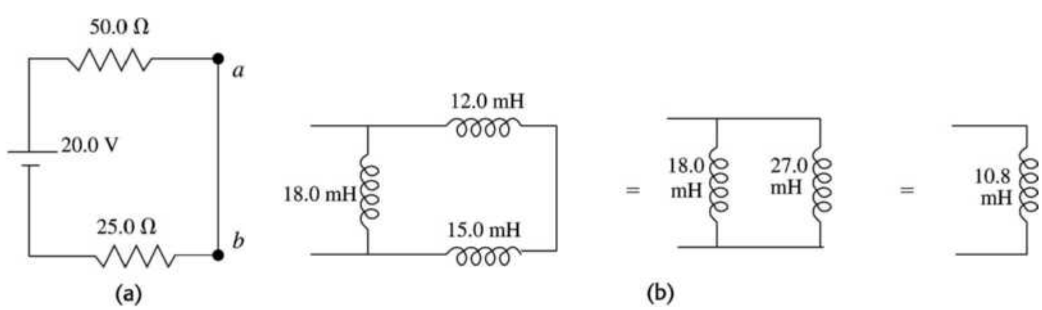


Figure 30.61

30.62. **IDENTIFY:** $i(t)$ is given by Eq. (30.14).

SET UP: The graph shows $V=0$ at $t=0$ and V approaches the constant value of 25 V at large times.

EXECUTE: **(a)** The voltage behaves the same as the current. Since V_R is proportional to i, the scope must be across the 150-Ω resistor.

(b) From the graph, as $t\to\infty, V_R\to25\text{ V}$, so there is no voltage drop across the inductor, so its internal resistance must be zero. $V_R=V_{max}(1-e^{-t/r})$. When $t=\tau$, $V_R=V_{max}\left(1-\frac{1}{e}\right)\approx0.63V_{max}$. From the graph, $V=0.63V_{max}=16\text{ V}$ at $t\approx0.5\text{ ms}$. Therefore $\tau=0.5\text{ ms}$. $L/R=0.5\text{ ms}$ gives $L=(0.5\text{ ms})(150\;\Omega)=0.075\text{ H}$.

(c) The graph if the scope is across the inductor is sketched in Figure 30.62

EVALUATE: At all times $V_R + V_L = 25.0$ V. At $t = 0$ all the battery voltage appears across the inductor since $i = 0$. At $t \to \infty$ all the battery voltage is across the resistance, since $di/dt = 0$.

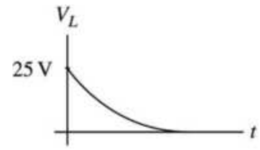


Figure 30.62

30.63. **IDENTIFY** and **SET UP:** The current grows in the circuit as given by Eq. (30.14). In an R-L circuit the full emf initially is across the inductance and after a long time is totally across the resistance. A solenoid in a circuit is represented as a resistance in series with an inductance. Apply the loop rule to the circuit; the voltage across a resistance is given by Ohm's law.

EXECUTE: **(a)** In the R-L circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. In the graph, the voltage drops, so the oscilloscope is across the solenoid.

(b) At $t \to \infty$ the current in the circuit approaches its final, constant value. The voltage doesn't go to zero because the solenoid has some resistance R_L. The final voltage across the solenoid is IR_L, where I is the final current in the circuit.

(c) The emf of the battery is the initial voltage across the inductor, 50 V. Just after the switch is closed, the current is zero and there is no voltage drop across any of the resistance in the circuit.

(d) As $t \to \infty, \mathcal{E} - IR - IR_L = 0$

$\mathcal{E} = 50$ V and from the graph $IR_L = 15$ V (the final voltage across the inductor), so

$IR_L = 35$ V and $I = (35 \text{ V})/R = 3.5$ A

(e) $IR_L = 15$ V, so $R_L = (15 \text{ V})/(3.5 \text{ A}) = 4.3\ \Omega$

$\mathcal{E} - V_L - iR = 0$, where V_L includes the voltage across the resistance of the solenoid.

$$V_L = \mathcal{E} - iR,\ i = \frac{\mathcal{E}}{R_{\text{tot}}}(1 - e^{-t/\tau}), \text{ so } V_L = \mathcal{E}\left[1 - \frac{R}{R_{\text{tot}}}(1 - e^{-t/\tau})\right]$$

$\mathcal{E} = 50$ V, $R = 10\ \Omega$, $R_{\text{tot}} = 14.3\ \Omega$, so when $t = \tau$, $V_L = 27.9$ V. From the graph, V_L has this value when $t = 3.0$ ms (read approximately from the graph), so $\tau = L/R_{\text{tot}} = 3.0$ ms. Then $L = (3.0 \text{ ms})(14.3\ \Omega) = 43$ mH.

EVALUATE: At $t = 0$ there is no current and the 50 V measured by the oscilloscope is the induced emf due to the inductance of the solenoid. As the current grows, there are voltage drops across the two resistances in the circuit. We derived an equation for V_L, the voltage across the solenoid. At $t = 0$ it gives $V_L = \mathcal{E}$ and at $t \to \infty$ it gives $V_L = \mathcal{E}R/R_{\text{tot}} = iR$.

30.64. **IDENTIFY:** At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in Figure 30.64a. $i = \dfrac{\mathcal{E}}{R} = \dfrac{50 \text{ V}}{150\ \Omega} = 0.333$ A. $V_1 = (100\ \Omega)(0.333 \text{ A}) = 33.3$ V.

$V_4 = (50\ \Omega)(0.333 \text{ A}) = 16.7$ V. $V_3 = 0$ since no current flows through it. $V_2 = V_4 = 16.7$ V, since the inductor is in parallel with the 50-Ω resistor. $A_1 = A_3 = 0.333$ A, $A_2 = 0$.

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.64b. $i = \mathcal{E}/R = \dfrac{50 \text{ V}}{130\ \Omega} = 0.385$ A.

$V_1 = (100\ \Omega)(0.385 \text{ A}) = 38.5$ V; $V_2 = 0$; $V_3 = V_4 = 50 \text{ V} - 38.5 \text{ V} = 11.5$ V.

$i_1 = 0.385$ A; $i_2 = \dfrac{11.5 \text{ V}}{75\ \Omega} = 0.153$ A; $i_3 = \dfrac{11.5 \text{ V}}{50\ \Omega} = 0.230$ A.

EVALUATE: Just after the switch is closed the current through the battery is 0.333 A. After a long time the current through the battery is 0.385 A. After a long time there is an additional current path, the equivalent resistance of the circuit is decreased and the current has increased.

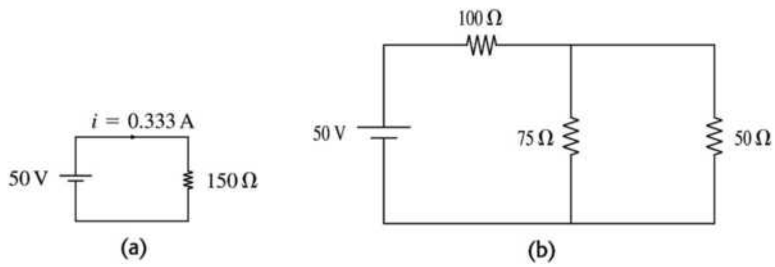


Figure 30.64

30.65. **IDENTIFY** and **SET UP:** Just after the switch is closed, the current in each branch containing an inductor is zero and the voltage across any capacitor is zero. The inductors can be treated as breaks in the circuit and the capacitors can be replaced by wires. After a long time there is no voltage across each inductor and no current in any branch containing a capacitor. The inductors can be replaced by wires and the capacitors by breaks in the circuit.

EXECUTE: **(a)** Just after the switch is closed the voltage V_5 across the capacitor is zero and there is also no current through the inductor, so $V_3 = 0$. $V_2 + V_3 = V_4 = V_5$, and since $V_5 = 0$ and $V_3 = 0, V_4$ and V_2 are also zero. $V_4 = 0$ means V_3 reads zero. V_1 then must equal 40.0 V, and this means the current read by A_1 is $(40.0\text{ V})/(50.0\ \Omega) = 0.800$ A. $A_2 + A_3 + A_4 = A_1$, but $A_2 = A_3 = 0$ so $A_4 = A_1 = 0.800$ A. $A_1 = A_4 = 0.800$ A; all other ammeters read zero. $V_1 = 40.0$ V and all other voltmeters read zero.

(b) After a long time the capacitor is fully charged so $A_4 = 0$, The current through the inductor isn't changing, so $V_2 = 0$. The currents can be calculated from the equivalent circuit that replaces the inductor by a short circuit, as shown in Figure 30.65a.

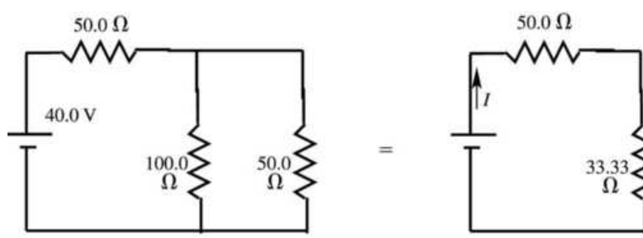


Figure 30.65a

$I = (40.0\text{ V})/(83.33\ \Omega) = 0.480\text{ A}$; A_1 reads 0.480 A

$V_1 = I(50.0\ \Omega) = 24.0\text{ V}$

The voltage across each parallel branch is $40.0\text{ V} - 24.0\text{ V} = 16.0\text{ V}$.

$V_2 = 0, V_3 = V_4 = V_5 = 16.0\text{ V}$

$V_3 = 16.0$ V means A_2 reads 0.160 A. $V_4 = 16.0$ V means A_3 reads 0.320 A. A_4 reads zero. Note that $A_2 + A_3 = A_1$.

(c) $V_5 = 16.0$ V so $Q = CV = (12.0\ \mu\text{F})(16.0\text{ V}) = 192\ \mu\text{C}$

(d) At $t = 0$ and $t \to \infty, V_2 = 0$. As the current in this branch increases from zero to 0.160 A the voltage V_2 reflects the rate of change of the current. The graph is sketched in Figure 30.65b.

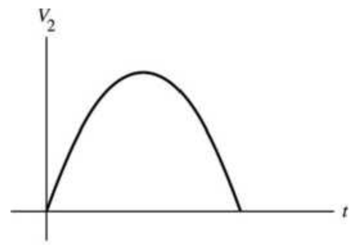


Figure 30.65b

EVALUATE: This reduction of the circuit to resistor networks only apply at $t = 0$ and $t \to \infty$. At intermediate times the analysis is complicated.

30.66. **IDENTIFY:** At all times $v_1 + v_2 = 25.0\text{ V}$. The voltage across the resistor depends on the current through it and the voltage across the inductor depends on the rate at which the current through it is changing.

SET UP: Immediately after closing the switch the current through the inductor is zero. After a long time the current is no longer changing.

EXECUTE: **(a)** $i = 0$ so $v_1 = 0$ and $v_2 = 25.0\text{ V}$. The ammeter reading is $A = 0$.

(b) After a long time, $v_2 = 0$ and $v_1 = 25.0\text{ V}$. $v_1 = iR$ and $i = \dfrac{v_1}{R} = \dfrac{25.0\text{ V}}{15.0\,\Omega} = 1.67\text{ A}$. The ammeter reading is $A = 1.67\text{A}$.

(c) None of the answers in (a) and (b) depend on L so none of them would change.

EVALUATE: The inductance L of the circuit affects the rate at which current reaches its final value. But after a long time the inductor doesn't affect the circuit and the final current does not depend on L.

30.67. **IDENTIFY:** At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\text{-}\Omega$ and $15.0\text{-}\Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\,\Omega + 15.0\,\Omega) = 0.455\text{ A}$. $A_1 = A_4 = 0.455\text{ A}$; $A_2 = A_3 = 0$.

(b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit. The circuit is equivalent to the circuit sketched in Figure 30.67. $I = (25.0\text{ V})/(42.73\,\Omega) = 0.585\text{ A}$. A_1 reads 0.585 A. The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\,\Omega) = 1.60\text{ V}$. A_2 reads $(1.60\text{ V})/(5.0\,\Omega) = 0.320\text{ A}$. A_3 reads $(1.60\text{ V})/(10.0\,\Omega) = 0.160\text{ A}$. A_4 reads $(1.60\text{ V})/(15.0\,\Omega) = 0.107\text{ A}$.

EVALUATE: Just after the switch is closed the current through the battery is 0.455 A. After a long time the current through the battery is 0.585 A. After a long time there are additional current paths, the equivalent resistance of the circuit is decreased and the current has increased.

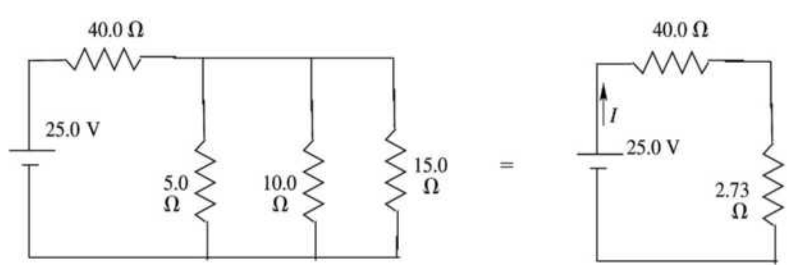


Figure 30.67

30.68. **IDENTIFY:** Closing S_2 and simultaneously opening S_1 produces an L-C circuit with initial current through the inductor of 3.50 A. When the current is a maximum the charge q on the capacitor is zero and when the charge q is a maximum the current is zero. Conservation of energy says that the maximum energy $\frac{1}{2}Li_{max}^2$ stored in the inductor equals the maximum energy $\frac{1}{2}\frac{q_{max}^2}{C}$ stored in the capacitor.

SET UP: $i_{max} = 3.50$ A, the current in the inductor just after the switch is closed.

EXECUTE: **(a)** $\frac{1}{2}Li_{max}^2 = \frac{1}{2}\frac{q_{max}^2}{C}$.

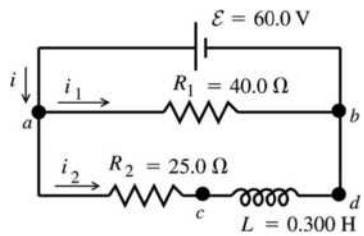
$q_{max} = (\sqrt{LC})i_{max} = \sqrt{(2.0\times10^{-3}\text{ H})(5.0\times10^{-6}\text{ F})}(3.50\text{ A}) = 3.50\times10^{-4}\text{ C} = 0.350\text{ mC}.$

(b) When q is maximum, $i = 0$.

EVALUATE: In the final circuit the current will oscillate.

30.69. **IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.69a.



The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0$.

Figure 30.69a

EXECUTE : **(a)** $\mathcal{E} - v_{ab} = 0$, so $v_{ab} = 60.0$ V

(b) The voltage drops across R, as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2R_2 = 0$

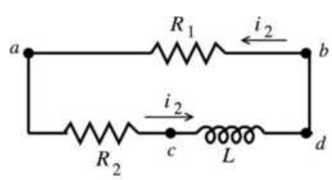
$\mathcal{E} - v_{R_2} - v_L = 0$ so $v_L = \mathcal{E} = 60.0$ V

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d.

(e) After the switch has been closed a long time, $\frac{di_2}{dt} \to 0$ so $v_L = 0$. Then $\mathcal{E} - v_{R_2} = 0$ and $i_2R_2 = \mathcal{E}$

so $i_2 = \frac{\mathcal{E}}{R_2} = \frac{60.0\text{ V}}{25.0\ \Omega} = 2.40$ A.

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40$ A. The circuit is sketched in Figure 30.69b.



EXECUTE: The current through R_1 is $i_2 = 2.40$ A in the direction b to a.

Thus $v_{ab} = -i_2R_1 = -(2.40\text{ A})(40.0\ \Omega)$

$v_{ab} = -96.0$ V.

Figure 30.69b

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0$

$v_L = v_{R_1} + v_{R_2}$

$v_{R_1} = -v_{ab} = 96.0\text{ V}$; $v_{R_2} = i_2 R_2 = (2.40\text{ A})(25.0\ \Omega) = 60.0\text{ V}$

Then $v_L = v_{R_1} + v_{R_2} = 96.0\text{ V} + 60.0\text{ V} = 156\text{ V}$.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point *d* is at higher potential than point *c*. The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -156\text{ V}$.

(h) Point *d* is at higher potential.

EVALUATE: The voltage across R_1 is constant once the switch is closed. In the branch containing R_2, just after S is closed the voltage drop is all across *L* and after a long time it is all across R_2. Just after S is opened the same current flows in the single loop as had been flowing through the inductor and the sum of the voltage across the resistors equals the voltage across the inductor. This voltage dies away, as the energy stored in the inductor is dissipated in the resistors.

30.70. **IDENTIFY:** Apply the loop rule to the two loops. The current through the inductor doesn't change abruptly.

SET UP: For the inductor $|\mathcal{E}| = L\left|\dfrac{di}{dt}\right|$ and $\mathcal{E}$ is directed to oppose the change in current.

EXECUTE: **(a)** Switch is closed, then at some later time

$$\frac{di}{dt} = 50.0\text{ A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\text{ H})(50.0\text{ A/s}) = 15.0\text{ V}.$$

The top circuit loop: $60.0\text{ V} = i_1 R_1 \Rightarrow i_1 = \dfrac{60.0\text{ V}}{40.0\ \Omega} = 1.50\text{ A}.$

The bottom loop: $60.0\text{ V} - i_2 R_2 - 15.0\text{ V} = 0 \Rightarrow i_2 = \dfrac{45.0\text{ V}}{25.0\ \Omega} = 1.80\text{ A}.$

(b) After a long time: $i_2 = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A}$, and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40\text{ A}$.

EVALUATE: The current through R_1 changes abruptly when the switch is closed.

30.71. **IDENTIFY** and **SET UP:** The circuit is sketched in Figure 30.71a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time *i* has reached its final value and $di/dt = 0$. The voltage across a resistor depends on *i* and the voltage across an inductor depends on di/dt.

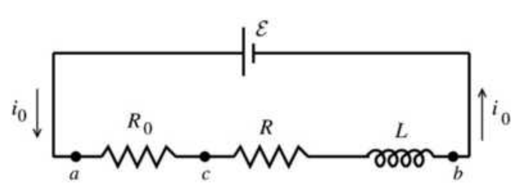


Figure 30.71a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$ so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0\text{ V}$. ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\dfrac{di_0}{dt} \to 0$ so the potential $-L\dfrac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0$.

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0 \text{ V}}{50.0\ \Omega + 150\ \Omega} = 0.180 \text{ A}$$

$$v_{ac} = i_0 R_0 = (0.180 \text{ A})(50.0\ \Omega) = 9.0 \text{ V}$$

Thus $v_{cb} = i_0 R + L\dfrac{di_0}{dt} = (0.180 \text{ A})(150\ \Omega) + 0 = 27.0 \text{ V}$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0$

$$\mathcal{E} - iR_0 - iR - L\frac{di}{dt} = 0$$

$L\dfrac{di}{dt} = \mathcal{E} - i(R_0 + R)$ and $\left(\dfrac{L}{R + R_0}\right)\dfrac{di}{dt} = -i + \dfrac{\mathcal{E}}{R + R_0}$

$$\frac{di}{-i + \mathcal{E}/(R + R_0)} = \left(\frac{R + R_0}{L}\right)dt$$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$:

$$\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R + R_0)} = \frac{R + R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R + R_0}\right]_0^{i_0} = \left(\frac{R + R_0}{L}\right)t, \text{ so}$$

$$\ln\left(-i_0 + \frac{\mathcal{E}}{R + R_0}\right) - \ln\left(\frac{\mathcal{E}}{R + R_0}\right) = -\left(\frac{R + R_0}{L}\right)t$$

$$\ln\left(\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)}\right) = -\left(\frac{R + R_0}{L}\right)t$$

Taking exponentials of both sides gives $\dfrac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} = e^{-(R+R_0)t/L}$ and $i_0 = \dfrac{\mathcal{E}}{R + R_0}(1 - e^{-(R+R_0)t/L})$.

Substituting in the numerical values gives $i_0 = \dfrac{36.0 \text{ V}}{50\ \Omega + 150\ \Omega}(1 - e^{-(200\ \Omega/4.00 \text{ H})t}) = (0.180 \text{ A})(1 - e^{-t/0.020 \text{ s}})$.

At $t \to 0, i_0 = (0.180 \text{ A})(1 - 1) = 0$ (agrees with part (a)). At $t \to \infty, i_0 = (0.180 \text{ A})(1 - 0) = 0.180 \text{ A}$ (agrees with part (b)).

$$v_{ac} = i_0 R_0 = \frac{\mathcal{E}R_0}{R + R_0}(1 - e^{-(R+R_0)t/L}) = 9.0 \text{ V}(1 - e^{-t/0.020 \text{ s}})$$

$$v_{cb} = \mathcal{E} - v_{ac} = 36.0 \text{ V} - 9.0 \text{ V}(1 - e^{-t/0.020 \text{ s}}) = 9.0 \text{ V}(3.00 + e^{-t/0.020 \text{ s}})$$

At $t \to 0, v_{ac} = 0, v_{cb} = 36.0$ V (agrees with part (a)). At $t \to \infty, v_{ac} = 9.0$ V, $v_{cb} = 27.0$ V (agrees with part (b)). The graphs are given in Figure 30.71b.

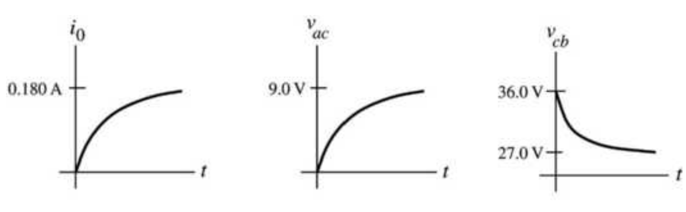


Figure 30.71b

EVALUATE: The expression for $i(t)$ we derived becomes Eq. (30.14) if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.72. IDENTIFY: Apply the loop rule. The current through the inductor doesn't change abruptly.

SET UP: With S_2 closed, v_{cb} must be zero.

EXECUTE: **(a)** Immediately after S_2 is closed, the inductor maintains the current $i = 0.180$ A through R. The loop rule around the outside of the circuit yields

$\mathcal{E} + \mathcal{E}_L - iR - i_0 R_0 = 36.0\text{ V} + (0.18\text{ A})(150\ \Omega) - (0.18\text{ A})(150\ \Omega) - i_0(50\ \Omega) = 0.$ $i_0 = \dfrac{36\text{ V}}{50\ \Omega} = 0.720\text{ A}.$

$v_{ac} = (0.72\text{ A})(50\ \Omega) = 36.0\text{ V}$ and $v_{cb} = 0.$

(b) After a long time, $v_{ac} = 36.0\text{ V},$ and $v_{cb} = 0.$ Thus $i_0 = \dfrac{\mathcal{E}}{R_0} = \dfrac{36.0\text{ V}}{50\ \Omega} = 0.720\text{ A},$ $i_R = 0$ and $i_{s2} = 0.720\text{ A}.$

(c) $i_0 = 0.720\text{ A},$ $i_R(t) = \dfrac{\mathcal{E}}{R_{\text{total}}} e^{-(R/L)t}$ and $i_R(t) = (0.180\text{ A})e^{-(12.5\text{ s}^{-1})t}.$

$i_{s2}(t) = (0.720\text{ A}) - (0.180\text{ A})e^{-(12.5\text{ s}^{-1})t} = (0.180\text{ A})(4 - e^{-(12.5\text{ s}^{-1})t}).$ The graphs of the currents are given in Figure 30.72.

EVALUATE: R_0 is in a loop that contains just $\mathcal{E}$ and $R_0,$ so the current through R_0 is constant. After a long time the current through the inductor isn't changing and the voltage across the inductor is zero. Since v_{cb} is zero, the voltage across R must be zero and i_R becomes zero.

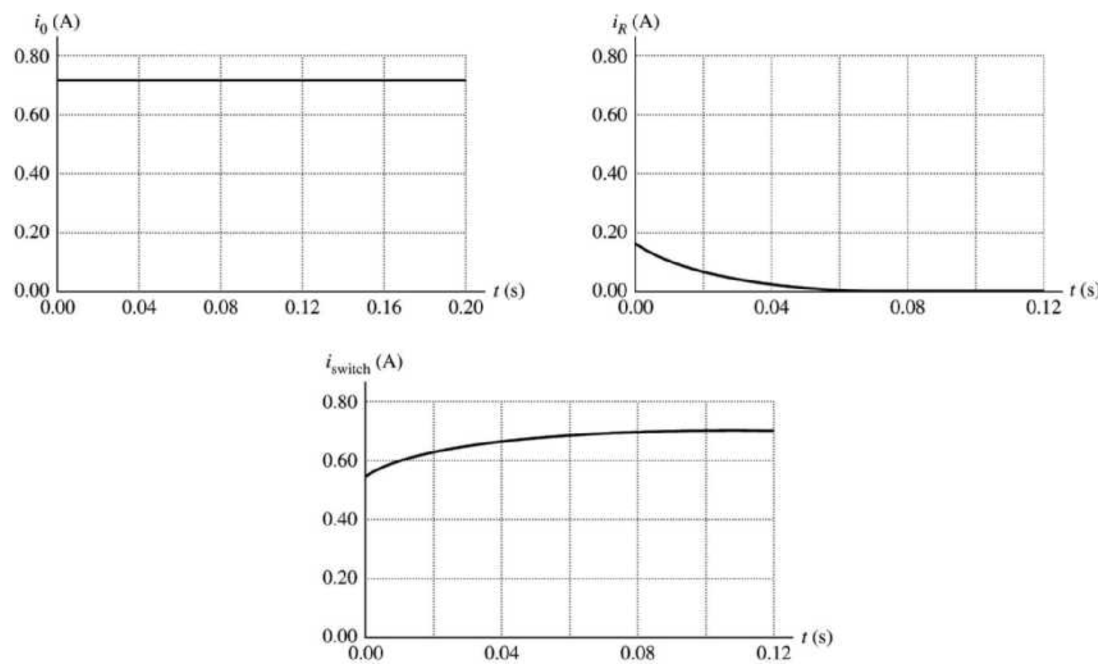


Figure 30.72

30.73. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: Find the flux through a ring of height h, radius r and thickness dr. Example 28.10 shows that $B = \dfrac{\mu_0 N i}{2\pi r}$ inside the toroid.

EXECUTE: **(a)** $\Phi_B = \int_a^b B(h\,dr) = \int_a^b \left(\dfrac{\mu_0 N i}{2\pi r}\right)(h\,dr) = \dfrac{\mu_0 N i h}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 N i h}{2\pi}\ln(b/a).$

(b) $L = \dfrac{N\Phi_B}{i} = \dfrac{\mu_0 N^2 h}{2\pi}\ln(b/a).$

(c) $\ln(b/a) = \ln(1-(b-a)/a) \approx \frac{b-a}{a} + \frac{(b-a)^2}{2a^2} + \cdots \Rightarrow L \approx \frac{\mu_0 N^2 h}{2\pi}\left(\frac{b-a}{a}\right).$

EVALUATE: $h(b-a)$ is the cross-sectional area A of the toroid and a is approximately the radius r, so this result is approximately the same as the result derived in Example 30.3.

30.74. **IDENTIFY:** At steady state with the switch in position 1, no current flows to the capacitors and the inductors can be replaced by wires. Apply conservation of energy to the circuit with the switch in position 2.

SET UP: Replace the series combinations of inductors and capacitors by their equivalents.

EXECUTE: **(a)** At steady state $i = \frac{\mathcal{E}}{R} = \frac{75.0\text{ V}}{125\ \Omega} = 0.600\text{ A}.$

(b) The equivalent circuit capacitance of the two capacitors is given by $\frac{1}{C_s} = \frac{1}{25\ \mu\text{F}} + \frac{1}{35\ \mu\text{F}}$ and

$C_s = 14.6\ \mu\text{F}$. $L_s = 15.0\text{ mH} + 5.0\text{ mH} = 20.0\text{ mH}$. The equivalent circuit is sketched in Figure 30.74a.

Energy conservation: $\frac{q^2}{2C} = \frac{1}{2}Li_0^2$. $q = i_0\sqrt{LC} = (0.600\text{ A})\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 3.24\times10^{-4}\text{ C}.$

As shown in Figure 30.74b, the capacitors have their maximum charge at $t = T/4$.

$t = \frac{1}{4}T = \frac{1}{4}(2\pi\sqrt{LC}) = \frac{\pi}{2}\sqrt{LC} = \frac{\pi}{2}\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 8.49\times10^{-4}\text{ s}$

EVALUATE: With the switch closed the battery stores energy in the inductors. This then is the energy in the L-C circuit when the switch is in position 2.

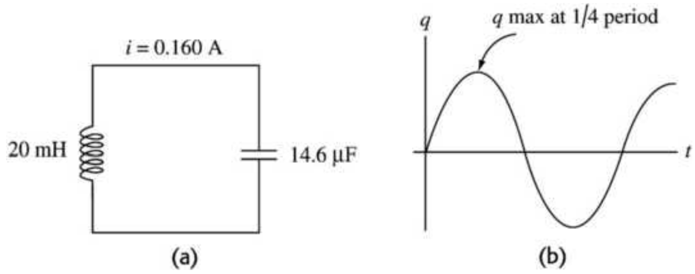


Figure 30.74

30.75. **(a) IDENTIFY** and **SET UP:** With switch S closed the circuit is shown in Figure 30.75a.

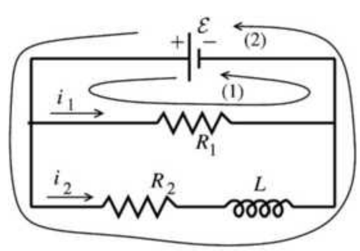


Figure 30.75a

Apply the loop rule to loops 1 and 2.

EXECUTE:

loop 1

$\mathcal{E} - i_1 R_1 = 0$

$i_1 = \frac{\mathcal{E}}{R_1}$ (independent of t)

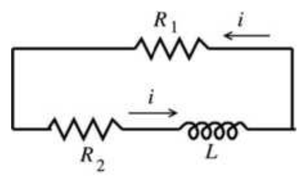
loop (2)

$\mathcal{E} - i_2 R_2 - L\frac{di_2}{dt} = 0$

This is in the form of Eq. (30.12), so the solution is analogous to Eq. (30.14): $i_2 = \frac{\mathcal{E}}{R_2}(1 - e^{-R_2 t/L})$

(b) EVALUATE: The expressions derived in part (a) give that as $t \to \infty, i_1 = \dfrac{\mathcal{E}}{R_1}$ and $i_2 = \dfrac{\mathcal{E}}{R_2}$. Since $\dfrac{di_2}{dt} \to 0$ at steady state, the inductance then has no effect on the circuit. The current in R_1 is constant; the current in R_2 starts at zero and rises to $\mathcal{E}/R_2$.

(c) IDENTIFY and SET UP: The circuit now is as shown in Figure 30.75b.



Let $t = 0$ now be when S is opened.

At $t = 0$, $i = \dfrac{\mathcal{E}}{R_2}$.

Figure 30.75b

Apply the loop rule to the single current loop.

EXECUTE: $-i(R_1 + R_2) - L\dfrac{di}{dt} = 0.$ (Now $\dfrac{di}{dt}$ is negative.)

$L\dfrac{di}{dt} = -i(R_1 + R_2)$ gives $\dfrac{di}{i} = -\left(\dfrac{R_1 + R_2}{L}\right)dt$

Integrate from $t = 0$, when $i = I_0 = \mathcal{E}/R_2$, to t.

$\int_{I_0}^{i} \dfrac{di}{i} = -\left(\dfrac{R_1 + R_2}{L}\right)\int_0^t dt$ and $\ln\left(\dfrac{i}{I_0}\right) = -\left(\dfrac{R_1 + R_2}{L}\right)t$

Taking exponentials of both sides of this equation gives $i = I_0 e^{-(R_1+R_2)t/L} = \dfrac{\mathcal{E}}{R_2}e^{-(R_1+R_2)t/L}$.

(d) IDENTIFY and SET UP: Use the equation derived in part (c) and solve for R_2 and $\mathcal{E}$.

EXECUTE: $L = 22.0$ H

$P_{R_1} = \dfrac{V^2}{R_1} = 40.0 \text{ W}$ gives $R_1 = \dfrac{V^2}{P_{R_1}} = \dfrac{(120 \text{ V})^2}{40.0 \text{ W}} = 360\ \Omega.$

We are asked to find R_2 and $\mathcal{E}$. Use the expression derived in part (c).

$I_0 = 0.600$ A so $\mathcal{E}/R_2 = 0.600$ A

$i = 0.150$ A when $t = 0.080$ s, so $i = \dfrac{\mathcal{E}}{R_2}e^{-(R_1+R_2)t/L}$ gives $0.150 \text{ A} = (0.600 \text{ A})e^{-(R_1+R_2)t/L}$

$\frac{1}{4} = e^{-(R_1+R_2)t/L}$ so $\ln 4 = (R_1 + R_2)t/L$

$R_2 = \dfrac{L\ln 4}{t} - R_1 = \dfrac{(22.0 \text{ H})\ln 4}{0.080 \text{ s}} - 360\ \Omega = 381.2\ \Omega - 360\ \Omega = 21.2\ \Omega$

Then $\mathcal{E} = (0.600 \text{ A})R_2 = (0.600 \text{ A})(21.2\ \Omega) = 12.7$ V.

(e) IDENTIFY and SET UP: Use the expressions derived in part (a).

EXECUTE: The current through the light bulb before the switch is opened is $i_1 = \dfrac{\mathcal{E}}{R_1} = \dfrac{12.7 \text{ V}}{360\ \Omega} = 0.0353$ A

EVALUATE: When the switch is opened the current through the light bulb jumps from 0.0353 A to 0.600 A. Since the electrical power dissipated in the bulb (brightness) depend on i^2, the bulb suddenly becomes much brighter.

30.76. IDENTIFY: Follow the steps specified in the problem.

SET UP: The current in an inductor does not change abruptly.

EXECUTE: **(a)** Using Kirchhoff's loop rule on the left and right branches:

Left: $\mathcal{E}-(i_1+i_2)R-L\dfrac{di_1}{dt}=0 \Rightarrow R(i_1+i_2)+L\dfrac{di_1}{dt}=\mathcal{E}.$

Right: $\mathcal{E}-(i_1+i_2)R-\dfrac{q_2}{C}=0 \Rightarrow R(i_1+i_2)+\dfrac{q_2}{C}=\mathcal{E}.$

(b) Initially, with the switch just closed, $i_1=0, i_2=\dfrac{\mathcal{E}}{R}$ and $q_2=0.$

(c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise but straightforward exercise. We will show that the initial conditions are satisfied: At $t=0, q_2=\dfrac{\mathcal{E}}{\omega R}e^{-\beta t}\sin(\omega t)=\dfrac{\mathcal{E}}{\omega R}\sin(0)=0.$

$i_1(t)=\dfrac{\mathcal{E}}{R}(1-e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow i_1(0)=\dfrac{\mathcal{E}}{R}(1-[\cos(0)])=0.$

(d) When does i_2 first equal zero? $\omega=\sqrt{\dfrac{1}{LC}-\dfrac{1}{(2RC)^2}}=625\text{ rad/s}.$

$i_2(t)=0=\dfrac{\mathcal{E}}{R}e^{-\beta t}[-(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t)+1=0$ and

$\tan(\omega t)=+2\omega RC=+2(625\text{ rad/s})(400\ \Omega)(2.00\times10^{-6}\text{ F})=+1.00.$

$\omega t=\arctan(+1.00)=+0.785 \Rightarrow t=\dfrac{0.785}{625\text{ rad/s}}=1.256\times10^{-3}\text{s}.$

EVALUATE: As $t\to\infty,\ i_1\to\mathcal{E}/R,\ q_2\to0$ and $i_2\to0.$

30.77. **IDENTIFY:** Apply $L=\dfrac{N\Phi_B}{i}$ to calculate L.

SET UP: In the air the magnetic field is $B_{\text{Air}}=\dfrac{\mu_0 Ni}{W}$. In the liquid, $B_{\text{L}}=\dfrac{\mu Ni}{W}.$

EXECUTE: **(a)** $\Phi_B=BA=B_{\text{L}}A_{\text{L}}+B_{\text{Air}}A_{\text{Air}}=\dfrac{\mu_0 Ni}{W}((D-d)W)+\dfrac{K\mu_0 Ni}{W}(dW)=\mu_0 Ni[(D-d)+Kd].$

$L=\dfrac{N\Phi_B}{i}=\mu_0N^2[(D-d)+Kd]=L_0-L_0\dfrac{d}{D}+L_{\text{f}}\dfrac{d}{D}=L_0+\left(\dfrac{L_{\text{f}}-L_0}{D}\right)d.$

$d=\left(\dfrac{L-L_0}{L_{\text{f}}-L_0}\right)D,$ where $L_0=\mu_0N^2D,$ and $L_{\text{f}}=K\mu_0N^2D.$

(b) and **(c)** Using $K=\chi_{\text{m}}+1$ we can find the inductance for any height $L=L_0\left(1+\chi_{\text{m}}\dfrac{d}{D}\right).$

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d=D/4$	0.63024 H	0.63000 H
$d=D/2$	0.63048 H	0.62999 H
$d=3D/4$	0.63072 H	0.62999 H
$d=D$	0.63096 H	0.62998 H

The values $\chi_{\text{m}}(O_2)=1.52\times10^{-3}$ and $\chi_{\text{m}}(\text{Hg})=-2.9\times10^{-5}$ have been used.

EVALUATE: **(d)** The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

30.78. **IDENTIFY:** The induced emf across the two coils is due to both the self-inductance of each and the mutual inductance of the pair of coils.

SET UP: The equivalent inductance is defined by $\mathcal{E}=L_{\text{eq}}\dfrac{di}{dt}$, where $\mathcal{E}$ and i are the total emf and current across the combination.

EXECUTE: Series: $L_1\frac{di_1}{dt}+L_2\frac{di_2}{dt}+M_{21}\frac{di_1}{dt}+M_{12}\frac{di_2}{dt}\equiv L_{\text{eq}}\frac{di}{dt}$.

But $i=i_1+i_2\Rightarrow\frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$ and $M_{12}=M_{21}\equiv M$, so $(L_1+L_2+2M)\frac{di}{dt}=L_{\text{eq}}\frac{di}{dt}$ and $L_{\text{eq}}=L_1+L_2+2M$.

Parallel: We have $L_1\frac{di_1}{dt}+M_{12}\frac{di_2}{dt}=L_{\text{eq}}\frac{di}{dt}$ and $L_2\frac{di_2}{dt}+M_{21}\frac{di_1}{dt}=L_{\text{eq}}\frac{di}{dt}$, with $\frac{di_1}{dt}+\frac{di_2}{dt}=\frac{di}{dt}$ and $M_{12}=M_{21}\equiv M$. To simplify the algebra let $A=\frac{di_1}{dt}$, $B=\frac{di_2}{dt}$, and $C=\frac{di}{dt}$. So $L_1A+MB=L_{\text{eq}}C,\ L_2B+MA=L_{\text{eq}}C,\ A+B=C$. Now solve for A and B in terms of C. $(L_1-M)A+(M-L_2)B=0$ using $A=C-B$. $(L_1-M)(C-B)+(M-L_2)B=0$. $(L_1-M)C-(L_1-M)B+(M-L_2)B=0$. $(2M-L_1-L_2)B=(M-L_1)C$ and $B=\frac{(M-L_1)}{(2M-L_1-L_2)}C$. But $A=C-B=C-\frac{(M-L_1)C}{(2M-L_1-L_2)}=\frac{(2M-L_1-L_2)-M+L_1}{(2M-L_1-L_2)}C$, or $A=\frac{M-L_2}{2M-L_1-L_2}C$. Substitute A in B back into original equation: $\frac{L_1(M-L_2)C}{2M-L_1-L_2}+\frac{M(M-L_1)}{(2M-L_1-L_2)}C=L_{\text{eq}}C$ and $\frac{M^2-L_1L_2}{2M-L_1-L_2}C=L_{\text{eq}}C$. Finally, $L_{\text{eq}}=\frac{L_1L_2-M^2}{L_1+L_2-2M}$.

EVALUATE: If the flux of one coil doesn't pass through the other coil, so $M=0$, then the results reduce to those of inductors in parallel.

30.79. **IDENTIFY:** Apply Kirchhoff's loop rule to the top and bottom branches of the circuit.

SET UP: Just after the switch is closed the current through the inductor is zero and the charge on the capacitor is zero.

EXECUTE: **(a)** $\mathcal{E}-i_1R_1-L\frac{di_1}{dt}=0\Rightarrow i_1=\frac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t})$.

$\mathcal{E}-i_2R_2-\frac{q_2}{C}=0\Rightarrow-\frac{di_2}{dt}R_2-\frac{i_2}{C}=0\Rightarrow i_2=\frac{\mathcal{E}}{R_2}e^{-(1/R_2C)t})$.

$q_2=\int_0^t i_2dt'=-\frac{\mathcal{E}}{R_2}R_2Ce^{-(1/R_2C)t'}\Big|_0^t=\mathcal{E}C(1-e^{-(1/R_2C)t})$.

(b) $i_1(0)\frac{\mathcal{E}}{R_1}(1-e^0)=0, i_2=\frac{\mathcal{E}}{R_2}e^0=\frac{48.0\text{ V}}{5000\ \Omega}=9.60\times10^{-3}\text{ A}$.

(c) As $t\to\infty$: $i_1(\infty)=\frac{\mathcal{E}}{R_1}(1-e^{-\infty})=\frac{\mathcal{E}}{R_1}=\frac{48.0\text{ V}}{25.0\ \Omega}=1.92\text{ A}$, $i_2=\frac{\mathcal{E}}{R_2}e^{-\infty}=0$. A good definition of a "long time" is many time constants later.

(d) $i_1=i_2\Rightarrow\frac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t})=\frac{\mathcal{E}}{R_2}e^{-(1/R_2C)t}\Rightarrow(1-e^{-(R_1/L)t})=\frac{R_1}{R_2}e^{-(1/R_2C)t}$. Expanding the exponentials like $e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$, we find: $\frac{R_1}{L}t-\frac{1}{2}\left(\frac{R_1}{L}\right)^2t^2+\cdots=\frac{R_1}{R_2}\left(1-\frac{t}{RC}+\frac{t^2}{2R^2C^2}-\cdots\right)$ and $t\left(\frac{R_1}{L}+\frac{R_1}{R_2{}^2C}\right)+O(t^2)+\cdots=\frac{R_1}{R_2}$, if we have assumed that $t<<1$. Therefore:

$$t\approx\frac{1}{R_2}\left(\frac{1}{(1/L)+(1/R_2^2C)}\right)=\left(\frac{LR_2C}{L+R_2^2C}\right)=\left(\frac{(8.0\text{ H})(5000\ \Omega)(2.0\times10^{-5}\text{ F})}{8.0\text{ H}+(5000\ \Omega)^2(2.0\times10^{-5}\text{ F})}\right)=1.6\times10^{-3}\text{s}.$$

(e) At $t = 1.57 \times 10^{-3}$ s: $i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \frac{48\text{ V}}{25\ \Omega}(1 - e^{-(25/8)t}) = 9.4 \times 10^{-3}$ A.

(f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960\text{ A} = i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = (1.92\text{ A})(1 - e^{-(R_1/L)t}).$$

$$e^{-(R_1/L)t} = 0.500 \Rightarrow t = -\frac{L}{R_1}\ln(0.5) = -\frac{8.0\text{ H}}{25\ \Omega}\ln(0.5) = 0.22\text{ s}.$$

EVALUATE: i_1 is initially zero and rises to a final value of 1.92 A. i_2 is initially 9.60 mA and falls to zero, q_2 is initially zero and rises to $q_2 = \mathcal{E}C = 960\ \mu$C.